## Algebra $\quad a,b,c,n,x\in\mathbb{R} \qquad i^2=-1$

| | |
|---|---|
| 1. Kvadratsetning: | $(a+b)^2=a^2+2ab+b^2$ |
| 2. Kvadratsetning: | $(a-b)^2=a^2-2ab+b^2$ |
| Konjugatsetningen: | $(a+b)(a-b)=a^2-b^2$ |
| Kvadratrotkonjugat: | $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})=a-b$ |
| Komplekskonjugat: | $(a+bi)(a-bi)=a^2+b^2$ |
| Andregradslikningen: | $ax^2+bx+c=0\Rightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ |
| Fullstendig kvadrat: | $ax^2+bx+c=a\cdot\left(x+\frac{b}{2a}\right)^2+c-\frac{b^2}{4a}$ |
| | $ax^{2n}+bx^n+c=a\cdot\left(x^n+\frac{b}{2a}\right)^2+c-\frac{b^2}{4a}$ |

## Trigonometriske identiteter $\quad x,a,b\in\mathbb{R} \quad n\in\mathbb{Z}$

$$\sin(x)^2+\cos(x)^2=1 \qquad \tan(x)=\frac{\sin(x)}{\cos(x)}$$

$$\sec(x)=1/\cos(x) \qquad \csc(x)=1/\sin(x) \qquad \cot(x)=1/\tan(x)$$

$$\sin(x\pm\pi/2)=\pm\cos(x) \qquad \sin(\pm\pi/2-x)=\pm\cos(x)$$
$$\cos(x\pm\pi/2)=\mp\sin(x) \qquad \cos(\pm\pi/2-x)=\pm\sin(x)$$
$$\tan(x\pm\pi/2)=-1/\tan(x) \qquad \tan(\pm\pi/2-x)=1/\tan(x)$$
$$\sin(x\pm3\pi/2)=\pm\cos(x) \qquad \sin(\pm3\pi/2-x)=\mp\cos(x)$$
$$\cos(x\pm3\pi/2)=\mp\sin(x) \qquad \cos(\pm3\pi/2-x)=\mp\sin(x)$$
$$\tan(x\pm3\pi/2)=-1/\tan(x) \qquad \tan(\pm3\pi/2-x)=1/\tan(x)$$
$$\sin(x+n\pi)=(-1)^n\sin(x) \qquad \sin(n\pi-x)=(-1)^{n+1}\sin(x)$$
$$\cos(x+n\pi)=(-1)^n\cos(x) \qquad \cos(n\pi-x)=(-1)^n\cos(x)$$
$$\tan(x+n\pi)=\tan(x) \qquad \tan(n\pi-x)=-\tan(x)$$
$$\sin(x+2n\pi)=\sin(x) \qquad \sin(2n\pi-x)=-\sin(x)$$
$$\cos(x+2n\pi)=\cos(x) \qquad \cos(2n\pi-x)=\cos(x)$$
$$\tan(x+2n\pi)=\tan(x) \qquad \tan(2n\pi-x)=-\tan(x)$$
$$\sin(n\pi)=0 \qquad \sin(-x)=-\sin(x)$$
$$\cos(n\pi)=(-1)^n \qquad \cos(-x)=\cos(x)$$
$$\tan(n\pi)=0 \qquad \tan(-x)=-\tan(x)$$

$$\sin(a\pm b)=\sin(a)\cdot\cos(b)\pm\cos(a)\cdot\sin(b)$$
$$\cos(a\pm b)=\cos(a)\cdot\cos(b)\mp\sin(a)\cdot\sin(b)$$
$$\tan(a\pm b)=\big(\tan(a)\pm\tan(b)\big)/\big(1\mp\tan(a)\cdot\tan(b)\big)$$

$$\sin(2x)=2\sin(x)\cdot\cos(x) \qquad \cos(2x)=2\cos(x)^2-1$$
$$\cos(2x)=\cos(x)^2-\sin(x)^2 \qquad \cos(2x)=1-2\sin(x)^2$$

## Potenser og røtter $\quad a,b,n,m\in\mathbb{R}^+ \quad k\in\mathbb{Z}^+$

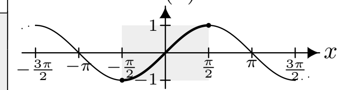
$$a^k=\overbrace{a\cdot a\cdots a}^{k\text{ faktorer}} \qquad \left(\frac{a}{b}\right)^n=\frac{a^n}{b^n} \qquad a^{1/n}=b>0 \text{ slik at } b^n=a$$
$$a^n=e^{n\cdot\ln(a)} \qquad 0^n=0 \qquad \sqrt[n]{a}=a^{1/n}$$
$$a^0=1 \qquad 0^{-n}=\frac{1}{0^n}=\frac{1}{0}=\varnothing \qquad \sqrt{a}=\sqrt[2]{a}=a^{1/2}$$
$$a^{-n}=\frac{1}{a^n} \qquad (-1)^k=+1\ (k\text{ partall}) \qquad \sqrt[n]{a^m}=a^{m/n}=(a^m)^{1/n}$$
$$a^{m+n}=a^m\cdot a^n \qquad (-1)^k=-1\ (k\text{ oddetall}) \qquad \sqrt[n]{a\cdot b}=\sqrt[n]{a}\cdot\sqrt[n]{b}$$
$$a^{m-n}=\frac{a^m}{a^n} \qquad (-a)^k=(-1)^k\cdot a^k \qquad \sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$
$$a^{m\cdot n}=(a^m)^n \qquad (-a)^{-k}=\frac{(-1)^k}{a^k} \qquad \sqrt[n]{-a}=-\sqrt[n]{a}\ (n\text{ oddetall})$$
$$(a\cdot b)^n=a^n\cdot b^n \qquad (-a)^0=1 \qquad \sqrt[n]{-a}\notin\mathbb{R}\ (n\text{ ikke oddetall})$$
$$0^0=1 \qquad \sqrt{a^2\cdot b}=a\cdot\sqrt{b}$$

## Logaritmer, fakultet $\quad r\in\mathbb{R} \quad b\neq1,x,y\in\mathbb{R}^+ \quad n,k\in\mathbb{N}$

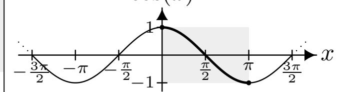
$$\log_b(x)=r \text{ slik at } b^r=x \qquad n!=1\cdot2\cdot3\cdots n$$
$$\ln(x)=\log_e(x) \qquad 0!=1$$
$$\log_b(x)=\frac{\ln(x)}{\ln(b)} \qquad \binom{n}{k}=\frac{n!}{(n-k)!\cdot k!}=\frac{n^{\underline{k}}}{k!}$$
$$\ln(x\cdot y)=\ln(x)+\ln(y) \qquad n^{\underline{k}}=n(n-1)\cdots(n-k+1)$$
$$\ln(x/y)=\ln(x)-\ln(y) \qquad n^{\underline{0}}=1$$
$$\ln(x^r)=r\cdot\ln(x)$$
$$\ln(e^r)=e^{\ln(r)}=r \qquad e\approx2.718281828459045$$
$$\ln(e)=1,\quad \ln(1)=0,\quad \ln(0)=\varnothing \qquad \pi\approx3.141592653589793$$

| $\theta$ | grader | $\cos(\theta)$ | $\sin(\theta)$ | $\tan(\theta)$ | $\theta-2\pi$ | grader |
|---|---|---|---|---|---|---|
| $0$ | $0°$ | $1$ | $0$ | $0$ | | |
| $\frac{\pi}{6}$ | $30°$ | $\frac{\sqrt3}{2}$ | $\frac12$ | $\frac{\sqrt3}{3}$ | | |
| $\frac{\pi}{4}$ | $45°$ | $\frac{\sqrt2}{2}$ | $\frac{\sqrt2}{2}$ | $1$ | | |
| $\frac{\pi}{3}$ | $60°$ | $\frac12$ | $\frac{\sqrt3}{2}$ | $\sqrt3$ | | |
| $\frac{\pi}{2}$ | $90°$ | $0$ | $1$ | $\pm\infty$ | | |
| $\frac{2\pi}{3}$ | $120°$ | $-\frac12$ | $\frac{\sqrt3}{2}$ | $-\sqrt3$ | | |
| $\frac{3\pi}{4}$ | $135°$ | $-\frac{\sqrt2}{2}$ | $\frac{\sqrt2}{2}$ | $-1$ | | |
| $\frac{5\pi}{6}$ | $150°$ | $-\frac{\sqrt3}{2}$ | $\frac12$ | $-\frac{\sqrt3}{3}$ | | |
| $\pi$ | $180°$ | $-1$ | $0$ | $0$ | | |
| $\frac{7\pi}{6}$ | $210°$ | $-\frac{\sqrt3}{2}$ | $-\frac12$ | $\frac{\sqrt3}{3}$ | | |
| $\frac{5\pi}{4}$ | $225°$ | $-\frac{\sqrt2}{2}$ | $-\frac{\sqrt2}{2}$ | $1$ | | |
| $\frac{4\pi}{3}$ | $240°$ | $-\frac12$ | $-\frac{\sqrt3}{2}$ | $\sqrt3$ | | |
| $\frac{3\pi}{2}$ | $270°$ | $0$ | $-1$ | $\pm\infty$ | $-\frac{\pi}{2}$ | $-90°$ |
| $\frac{5\pi}{3}$ | $300°$ | $\frac12$ | $-\frac{\sqrt3}{2}$ | $-\sqrt3$ | $-\frac{\pi}{3}$ | $-60°$ |
| $\frac{7\pi}{4}$ | $315°$ | $\frac{\sqrt2}{2}$ | $-\frac{\sqrt2}{2}$ | $-1$ | $-\frac{\pi}{4}$ | $-45°$ |
| $\frac{11\pi}{6}$ | $330°$ | $\frac{\sqrt3}{2}$ | $-\frac12$ | $-\frac{\sqrt3}{3}$ | $-\frac{\pi}{6}$ | $-30°$ |
| $2\pi$ | $360°$ | $1$ | $0$ | $0$ | $0$ | $0°$ |

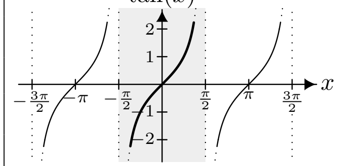
$\sin(x)$

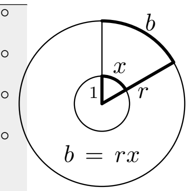



$\cos(x)$




$\tan(x)$




## Derivasjon $\quad$ mhp $x\in\mathbb{R}$ og $f,g$ er funksjoner av $x$

$$(a\cdot f)'=a\cdot f' \qquad (x^n)'=n\cdot x^{n-1} \qquad \sin(x)'=\cos(x)$$
$$(f+g)'=f'+g' \qquad (e^x)'=e^x \qquad \cos(x)'=-\sin(x)$$
$$(f\cdot g)'=f'\cdot g+f\cdot g' \qquad (a^x)'=a^x\cdot\ln(a) \qquad \tan(x)'=\frac{1}{\cos(x)^2}$$
$$\left(\tfrac{1}{f}\right)'=-\frac{f}{f^2} \qquad \ln(x)'=\frac{1}{x}\ (x>0) \qquad \sin^{-1}(x)'=\frac{1}{\sqrt{1-x^2}}$$
$$\left(\tfrac{f}{g}\right)'=\frac{f'\cdot g-f\cdot g'}{g^2} \qquad \log_b(x)'=\frac{1}{\ln(b)\cdot x} \qquad \cos^{-1}(x)'=\frac{-1}{\sqrt{1-x^2}}$$
$$f(g)'=f'(g)\cdot g' \qquad \ln(f)'=\frac{f'}{f}\ (f>0) \qquad \tan^{-1}(x)'=\frac{1}{1+x^2}$$

## Integrasjon $\quad$ mhp $x\in\mathbb{R}$ og $f,g$ er funksjoner av $x$

$$\int x^n\,\mathrm{d}x=\frac{1}{n+1}x^{n+1}+C \qquad \int\sin(x)\,\mathrm{d}x=-\cos(x)+C$$
$$\int e^x\,\mathrm{d}x=e^x+C \qquad \int\cos(x)\,\mathrm{d}x=\sin(x)+C$$
$$\int a^x\,\mathrm{d}x=\frac{1}{\ln(a)}a^x+C \qquad \int\tan(x)\,\mathrm{d}x=-\ln|\cos(x)|+C$$
$$\int\frac{1}{x}\,\mathrm{d}x=\ln|x|+C \qquad \int\frac{1}{\cos(x)^2}\,\mathrm{d}x=\tan(x)+C$$
$$\int\ln(x)\,\mathrm{d}x=x\cdot\ln(x)+C \qquad \int\frac{1}{\sqrt{a^2-x^2}}\,\mathrm{d}x=\sin^{-1}\left(\frac{x}{a}\right)+C$$
$$\int\frac{f'}{f}\,\mathrm{d}x=\ln|f|+C \qquad \int\frac{\mathrm{d}x}{a^2+x^2}=\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$$

$$\int f(ax+b)\,\mathrm{d}x=\frac{1}{a}f^{(-1)}(ax+b)+C$$

$$\int f\big(g(x)\big)\cdot g'(x)\,\mathrm{d}x=\int f(u)\,\mathrm{d}u=f^{(-1)}\big(g(x)\big) \qquad \big(u\leftrightarrow g(x)\big)$$
$$\int_a^b f\big(g(x)\big)\cdot g'(x)\,\mathrm{d}x=\int_{g(a)}^{g(b)} f(u)\,\mathrm{d}u \qquad \big(u\leftrightarrow g(x)\big)$$

$$\int f\cdot g\,\mathrm{d}x=f\cdot g^{(-1)}-\int f^{(1)}\cdot g^{(-1)}\,\mathrm{d}x$$
$$\int f\cdot g\,\mathrm{d}x=f\cdot g^{(-1)}-f^{(1)}\cdot g^{(-2)}+\int f^{(2)}\cdot g^{(-2)}\,\mathrm{d}x$$
$$\int f\cdot g\,\mathrm{d}x=f\cdot g^{(-1)}-f^{(1)}\cdot g^{(-2)}+f^{(2)}\cdot g^{(-3)}-f^{(3)}\cdot g^{(-4)}+\cdots$$
$$\cdots+(-1)^{n-1}f^{(n-1)}\cdot g^{(-n)}+(-1)^n\int f^{(n)}\cdot g^{(-n)}\,\mathrm{d}x$$

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