8.4.6

$$x = \cos(t) + t\cdot\sin(t)$$
$$y = \sin(t) - t\cdot\cos(t) \qquad (0 \leq t \leq 2\pi)$$

$$\left(\frac{dx}{dt}\right)^2 = \left(-\sin(t) + 1\cdot\sin(t) + t\cdot\cos(t)\right)^2 = \left(t\cdot\cos(t)\right)^2 = t^2\cdot\cos^2(t)$$

$$\left(\frac{dy}{dt}\right)^2 = \left(\cos(t) - \left(1\cdot\cos(t) + t\cdot(-\sin(t))\right)\right)^2$$
$$= \left(\cos(t) - \cos(t) + t\cdot\sin(t)\right)^2$$
$$= t^2\cdot\sin^2(t)$$

$$s = \int_0^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_0^{2\pi} \sqrt{t^2\cdot\cos^2(t) + t^2\cdot\sin^2(t)}\, dt$$

$$= \int_0^{2\pi} \sqrt{t^2\underbrace{\left(\sin^2(t) + \cos^2(t)\right)}_{1}}\, dt = \int_0^{2\pi} \sqrt{t^2\cdot 1}\, dt = \int_0^{2\pi} t\, dt$$

$$= \left[\frac{1}{2}t^2\right]_0^{2\pi} = \left(\frac{1}{2}\cdot(2\pi)^2\right) - \left(\frac{1}{2}\cdot 0^2\right)$$

$$= \frac{1}{2}\cdot 2^2\cdot\pi^2 = 2\pi^2$$

8.4.11

Øvt 4 2

$$S = 2\pi \int_a^b |g(t)| \cdot \sqrt{(f'(t))^2 + (g'(t))^2}\, dt$$

$$x = e^t \cdot \cos(t) = f(t) \qquad (0 \le t \le \tfrac{\pi}{2})$$
$$y = e^t \cdot \sin(t) = g(t)$$

$$\underset{a}{0} \qquad \underset{b}{\tfrac{\pi}{2}}$$

$$(f'(t))^2 = \left(e^t \cdot \cos(t) + e^t \cdot (-\sin(t))\right)^2$$
$$= \left(e^t \cdot \cos(t) - e^t \cdot \sin(t)\right)^2$$
$$= \left(e^t \cdot (\cos(t) - \sin(t))\right)^2$$
$$= (e^t)^2 \cdot (\cos(t) - \sin(t))^2$$

$$(f'(t))^2 = e^{2t} \cdot \left(\cos^2(t) - 2 \cdot \cos(t) \cdot \sin(t) + \sin^2(t)\right)$$

$$(g'(t))^2 = e^{2t} \cdot \left(\cos^2(t) + 2 \cdot \cos(t) \cdot \sin(t) + \sin^2(t)\right)$$

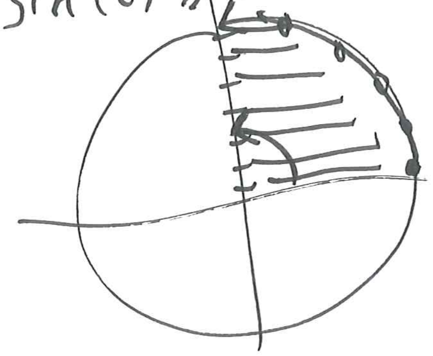
$$(f'(t))^2 + (g'(t))^2 = e^{2t}\left(\cos^2 t - \cancel{2\cos t \sin t} + \sin^2 t + \cos^2 t + \cancel{2\cos t \sin t} + \sin^2 t\right)$$
$$= e^{2t}(2\cos^2 t + 2\sin^2 t)$$
$$= 2 \cdot e^{2t}(\underbrace{\cos^2 t + \sin^2 t}_{1})$$
$$= 2 \cdot e^{2t}$$

Øvt 4 3

$$S = 2\pi \int_0^{\pi/2} \left| e^t \cdot \sin(t) \right| \cdot \sqrt{2 \cdot e^{2t}}\, dt$$

$e^t > 0$ når $t \in [0, \frac{\pi}{2}]$

$\sin(t) \geq 0$ når $t \in (0, \frac{\pi}{2}]$



$$S = 2\pi \int_0^{\pi/2} \underline{e^t} \cdot \underline{\sin(t)} \cdot \sqrt{2} \cdot \underline{\sqrt{(e^t)^2}}\, dt$$

$$= 2\pi \cdot \sqrt{2} \cdot \int_0^{\pi/2} e^t \cdot e^t \cdot \sin(t)\, dt$$

$$= 2\sqrt{2}\pi \int_0^{\pi/2} e^{2t} \cdot \sin(t) \cdot dt$$

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$$\int e^{at} \cdot \sin bt \, dt = \frac{e^{at}}{a^2+b^2} \cdot \left(a \cdot \sin bt - b \cdot \cos bt\right) + C$$

$a = 2 \qquad b = 1$

$$S = 2 \cdot \sqrt{2} \cdot \pi \cdot \left[\frac{e^{2t}}{5} \left(2\sin t - \cos t\right) \right]_0^{\pi/2}$$

$$= \frac{2}{5}\sqrt{2}\,\pi \cdot \left(\left(e^{\pi} \cdot \left(2 \cdot \sin\frac{\pi}{2} - \cos\frac{\pi}{2}\right) \right) - \left(e^{0}\left(2 \cdot \sin 0 - \cos 0\right) \right) \right)$$

$$= \frac{2}{5}\sqrt{2}\,\pi \cdot \left(\left(e^{\pi} \cdot \left(2 - 0 \right) \right) - \left(1 \cdot \left(2 \cdot 0 - 1 \right) \right) \right)$$

$$= \frac{2}{5}\sqrt{2}\,\pi \left(2e^{\pi} + 1 \right)$$