

10-2.2

Oct 8

①

$$\vec{u} = \vec{i} - \vec{j}, \quad \vec{v} = \vec{j} + 2\vec{k}$$

$$a) \quad \vec{u} + \vec{v} = \vec{i} - \vec{j} + \vec{j} + 2\vec{k} = \vec{i} + 2\vec{k}$$

$$\vec{u} - \vec{v} = \vec{i} - \vec{j} - (\vec{j} + 2\vec{k}) = \vec{i} - \vec{j} - \vec{j} - 2\vec{k} = \vec{i} - 2\vec{j} - 2\vec{k}$$

$$2\vec{u} - 3\vec{v} = 2(\vec{i} - \vec{j}) - 3(\vec{j} + 2\vec{k}) = 2\vec{i} - 2\vec{j} - 3\vec{j} - 6\vec{k} \\ = 2\vec{i} - 5\vec{j} - 6\vec{k}$$

$$b) \quad |\vec{u}| = \sqrt{1^2 + (-1)^2 + (0)^2} = \sqrt{2}$$

$$|\vec{v}| = \sqrt{1^2 + 2^2} = \sqrt{5}$$

$$c) \quad \hat{u} = \frac{1}{|\vec{u}|} \cdot \vec{u} = \frac{1}{\sqrt{2}} \cdot (\vec{i} - \vec{j})$$

$$\hat{v} = \frac{1}{|\vec{v}|} \cdot \vec{v} = \frac{1}{\sqrt{5}} \cdot (\vec{j} + 2\vec{k})$$

$$d) \quad \vec{u} \cdot \vec{v} = (\vec{i} - \vec{j}) \cdot (\vec{j} + 2\vec{k}) = 1 \cdot 0 + (-1) \cdot 1 + 0 \cdot 2 = -1$$

$$e) \quad \theta = \cos^{-1} \left(\frac{\vec{u} \cdot \vec{v}}{|\vec{u}| \cdot |\vec{v}|} \right) = \cos^{-1} \left(\frac{-1}{\sqrt{2} \cdot \sqrt{5}} \right) = \cos^{-1} \left(\frac{-1}{\sqrt{10}} \right)$$

$$= 1.89 \text{ radians}$$

$$= 108.4^\circ$$

$$f) \quad \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|} = \frac{-1}{\sqrt{5}}$$

$$g) \quad \vec{v}_{\vec{u}} = \frac{\vec{v} \cdot \vec{u}}{|\vec{u}|} \cdot \hat{u} = \frac{-1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \cdot (\vec{i} - \vec{j}) \\ = -\frac{1}{2}(\vec{i} - \vec{j})$$