

③ a) Kjeglesnitt

i)  $y^2 - 4y + 8 - x = 0$

$$x = y^2 - 4y + 8$$

Dette er en parabel med  
akse // med x-aksen:

Alse:  $y = -\frac{-4}{2 \cdot 1} = 2 = y_0$

Toppunkt:  $(x_0 = -\frac{(-4)^2}{4 \cdot 1} + 8 = 4, 2)$

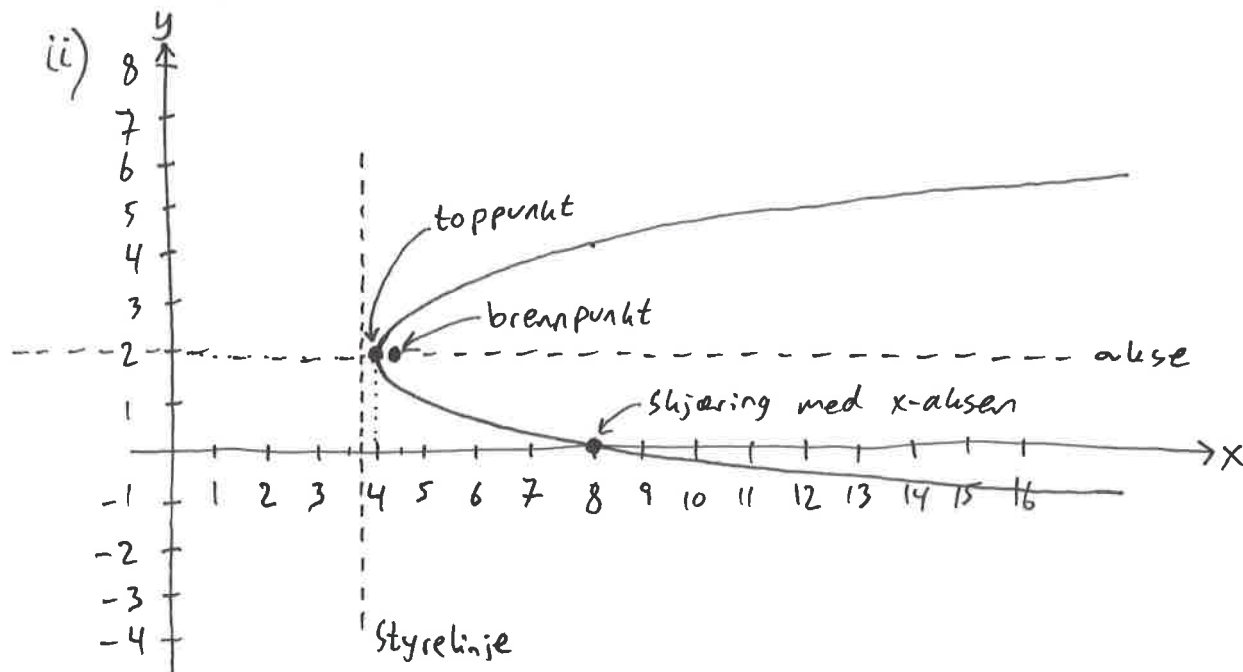
$$x = (y - 2)^2 + 4$$

Brennpunkt:  $(4 + \frac{1}{4 \cdot 1} = 4.25, 2)$

Styrelinje:  $x = 4 - \frac{1}{4 \cdot 1} = 3.75$

Nullpunkter:  $y = 2 \pm \sqrt{2^2 - \frac{8}{1}} = \emptyset$   
(ingen nullpunkter)

Skjæring med x-aksen:  $x = 8$



b) Linjer og afstande i rummet

i)  $(x, y, z) = (-3, -4, -5)$

$$\begin{cases} r = \sqrt{9+16} = 5 \\ \theta = \text{atan2}(-4, -3) = \tan^{-1}(-4/(-3)) - \pi = -2.214 \\ z = -5 \end{cases}$$

Sylindrisk:  $[r, \theta, z] = [5, -2.214, -5]$

$$\begin{cases} R = \sqrt{9+16+25} = 5\sqrt{2} \\ \phi = \cos^{-1}(5/5\sqrt{2}) = 3\pi/4 \\ \theta = -2.214 \end{cases}$$

Sfærisk:  $[R, \phi, \theta] = [5\sqrt{2}, 3\pi/4, -2.214]$

ii)  $l: \vec{r}_1 = \vec{i} - \vec{j} + 2\vec{k}, \vec{v}_1 = -2\vec{i} + \vec{j} + \vec{k}$

$m: \vec{r}_2 = 4\vec{i} + 5\vec{j}, \vec{v}_2 = 5\vec{j} - 2\vec{k}$

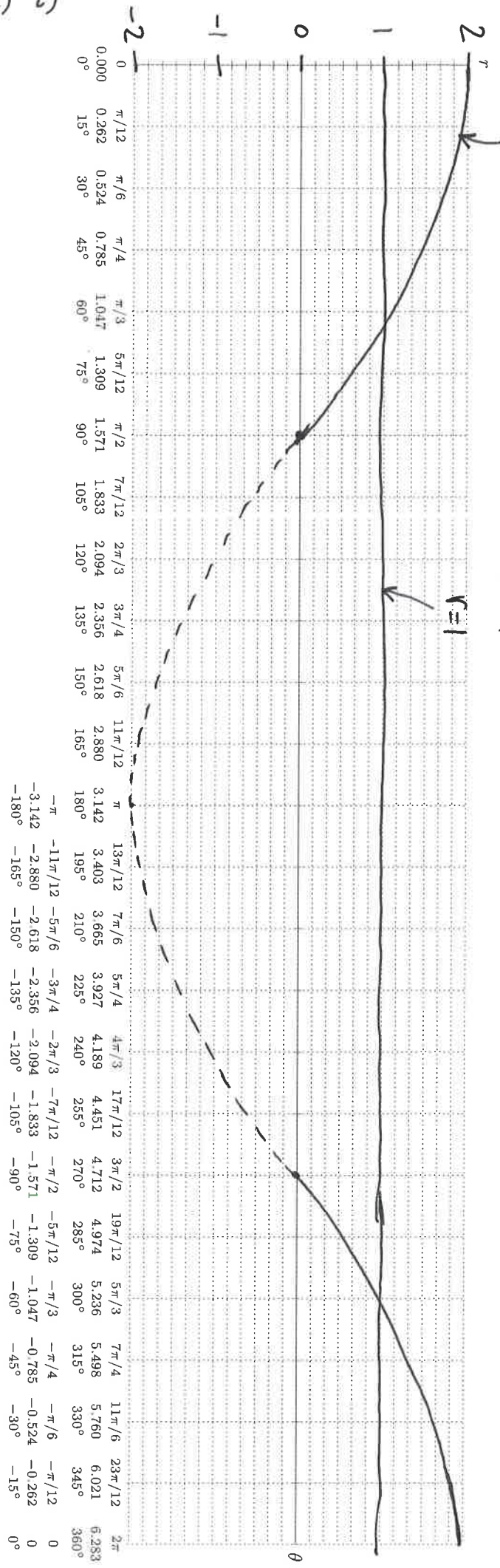
$$\vec{r}_2 - \vec{r}_1 = (4-1)\vec{i} + (5-(-1))\vec{j} + (0-2)\vec{k} = 3\vec{i} + 6\vec{j} - 2\vec{k}$$

$$\vec{v}_1 \times \vec{v}_2 = \begin{bmatrix} 1 & 1 & -2 & 1 \\ 5 & -2 & 0 & 5 \end{bmatrix} = -7\vec{i} - 4\vec{j} - 10\vec{k}$$

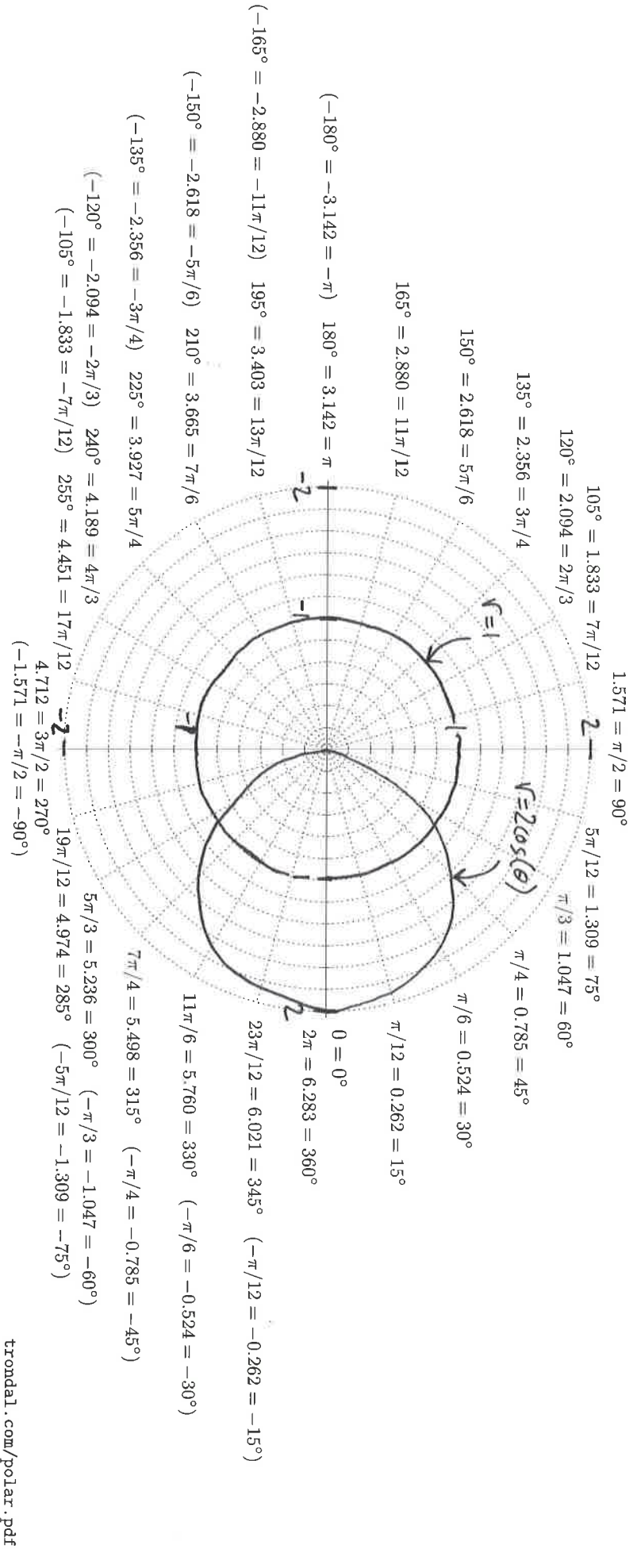
$$s = \frac{|(\vec{r}_2 - \vec{r}_1) \cdot (\vec{v}_1 \times \vec{v}_2)|}{|\vec{v}_1 \times \vec{v}_2|} = \frac{|(3\vec{i} + 6\vec{j} - 2\vec{k}) \cdot (-7\vec{i} - 4\vec{j} - 10\vec{k})|}{|-7\vec{i} - 4\vec{j} - 10\vec{k}|}$$

$$s = \frac{|-21 - 24 + 20|}{\sqrt{7^2 + 4^2 + 10^2}} = \frac{25}{\sqrt{165}} \approx 1.946$$

$r = 2\cos(\theta) = 0 \Rightarrow \cos(\theta) = 0 \Rightarrow \theta = \{ \pi/2, 3\pi/2 \}$



Løsningsforslag (3) c) i)



$\theta$	$r$
0°	2.000
15°	1.962
30°	1.924
45°	1.885
60°	1.847
75°	1.809
90°	1.571
105°	1.833
120°	2.094
135°	2.356
150°	2.618
165°	2.880
180°	3.142
195°	3.403
210°	3.665
225°	3.927
240°	4.189
255°	4.451
270°	4.712
285°	4.974
300°	5.236
315°	5.498
330°	5.760
345°	6.021
360°	6.283

- 165° = 2.880 = 11π/12
- 150° = 2.618 = 5π/6
- 135° = 2.356 = 3π/4
- 120° = 2.094 = 2π/3
- 105° = 1.833 = 7π/12
- 90° = 1.571 = π/2
- 75° = 1.309 = 5π/12
- 60° = 1.047 = π/3
- 45° = 0.785 = π/4
- 30° = 0.524 = π/6
- 15° = 0.262 = π/12
- 0 = 0°
- 2π = 6.283 = 360°
- 23π/12 = 6.021 = 345°
- (-π/12 = -0.262 = -15°)
- 11π/6 = 5.760 = 330°
- (-π/6 = -0.524 = -30°)
- 7π/4 = 5.498 = 315°
- (-π/4 = -0.785 = -45°)
- 5π/3 = 5.236 = 300°
- (-π/3 = -1.047 = -60°)
- 19π/12 = 4.974 = 285°
- (-5π/12 = -1.309 = -75°)
- 4.712 = 3π/2 = 270°
- (-1.571 = -π/2 = -90°)
- (-135° = -2.356 = -3π/4)
- 225° = 3.927 = 5π/4
- (-150° = -2.618 = -5π/6)
- 210° = 3.665 = 7π/6
- (-180° = -3.142 = -π)
- 180° = 3.142 = π
- (-165° = -2.880 = -11π/12)
- 195° = 3.403 = 13π/12
- (-120° = -2.094 = -2π/3)
- 240° = 4.189 = 4π/3
- (-105° = -1.833 = -7π/12)
- 255° = 4.451 = 17π/12
- (-90° = -1.571 = -π/2)
- 270° = 4.712 = 3π/2
- (-75° = -1.309 = -5π/12)
- 285° = 4.974 = 19π/12
- (-60° = -1.047 = -π/3)
- 300° = 5.236 = 5π/3
- (-45° = -0.785 = -π/4)
- 315° = 5.498 = 7π/4
- (-30° = -0.524 = -π/6)
- 330° = 5.760 = 11π/6
- (-15° = -0.262 = -π/12)
- 345° = 6.021 = 23π/12
- 360° = 6.283 = 2π

c) i) Se polar-ark.

ii)  $r=f(\theta) = 2\cos(\theta)$  og  $r=g(\theta) = 1$ , skjæringspunkter:

$$\begin{array}{ll} 1. & f(\theta) = 0 & g(\theta) = 0 \\ & 2\cos(\theta) = 0 & 1 = 0 \\ & \theta = \left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\} & \text{Ingen løsning.} \end{array}$$

$\Rightarrow$  ikke skjæring i origo.

$$2. \quad f(\theta) = g(\theta)$$

$$2\cos(\theta) = 1$$

$$\cos(\theta) = \frac{1}{2}$$

$$\theta = \left\{ \frac{\pi}{3}, \frac{5\pi}{3} \right\}$$

$\Rightarrow$  skjæringspunkter:

$$\left[ 1, \frac{\pi}{3} \right] \text{ og } \left[ 1, \frac{5\pi}{3} \right]$$

$$3. \quad f(\theta + (2k+1)\pi) = -g(\theta)$$

$$2\cos(\theta + 2k\pi + \pi) = -1$$

$$2\cos(\theta + \pi) = -1$$

$$\cos(\theta + \pi) = -\frac{1}{2}$$

$$\theta + \pi = \left\{ \frac{2\pi}{3}, \frac{4\pi}{3} \right\}$$

$$\theta = \left\{ -\frac{\pi}{3}, \frac{\pi}{3} \right\}$$

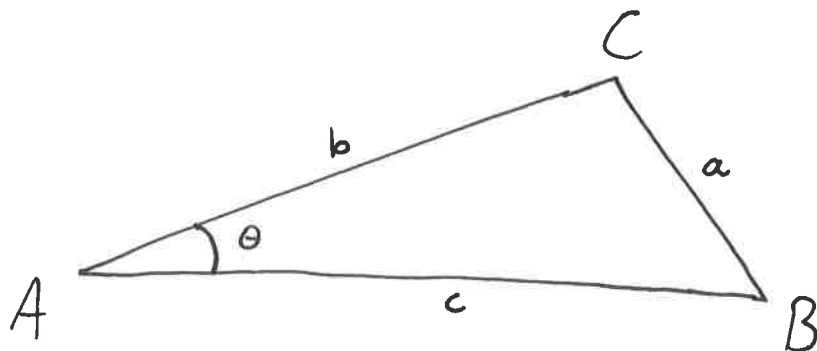
$\rightarrow$  samme som fra 2.

Skjæring i  $\left[ 1, -\frac{\pi}{3} \right] = \left[ 1, -\frac{\pi}{3} + 2\pi \right] = \left[ 1, \frac{5\pi}{3} \right]$  (samme som <sup>fra</sup> 2.)

$$d) \ i) \quad |\vec{AB}| = \sqrt{(1-2)^2 + (1-(-4))^2 + (5-2)^2} = \sqrt{35} = c$$

$$|\vec{AC}| = \sqrt{(0-2)^2 + (0-(-4))^2 + (3-2)^2} = \sqrt{21} = b$$

$$|\vec{BC}| = \sqrt{(0-1)^2 + (0-1)^2 + (3-5)^2} = \sqrt{6} = a$$



$$a^2 = b^2 + c^2 - 2bc \cdot \cos(\theta)$$

$$\theta = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right) = \cos^{-1}\left(\frac{21 + 35 - 6}{2 \cdot \sqrt{21} \cdot \sqrt{35}}\right) = \cos^{-1}\left(\frac{5\sqrt{15}}{21}\right)$$

$$\theta = \cos^{-1}\left(\frac{5\sqrt{15}}{21}\right) = 0.397 = 22.76^\circ$$

ii) Se neste arh.

d) (ii)

punktene er skjæringen mellom  
skallet/veggen til en sylinder  
med radius 3 ~~som~~ og senter  
langt z-aksen, og planet

$Z = 0$ . Dette blir en  
sirkel i xy-planet med  
senter i origo og radius 3.

