Eksamen 14. mai 2014

(3) a) Kjeglesnitt

i) $y^2 - 4y + 8 - x = 0$

$x = y^2 - 4y + 8$

Dette er en parabel med akse $\parallel$ med x-aksen:

Akse: $y = -\frac{-4}{2\cdot 1} = 2 = y_0$

Toppunkt: $\left(x_0 = -\frac{(-4)^2}{4\cdot 1} + 8 = 4, 2\right)$

$x = (y-2)^2 + 4$

Brennpunkt: $\left(4 + \frac{1}{4\cdot 1} = 4.25, 2\right)$

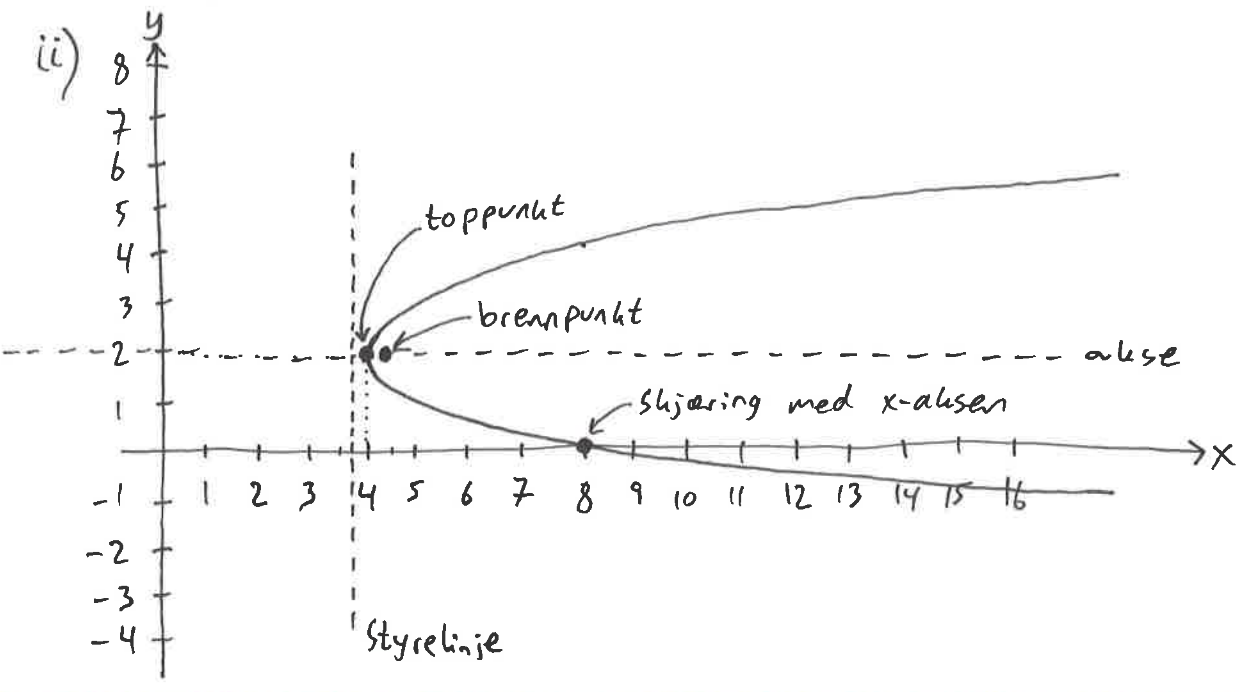
Styrelinje: $x = 4 - \frac{1}{4\cdot 1} = 3.75$

Nullpunkter: $y = 2 \pm \sqrt{2^2 - \frac{8}{1}} = \emptyset$

(ingen nullpunkter)

Skjæring med x-aksen: $x = 8$

ii)



b) Linjer og avstander i rommet

i) $(x, y, z) = (-3, -4, -5)$

$$\begin{cases} r = \sqrt{9+16} = 5 \\ \theta = \operatorname{atan2}(-4,-3) = \tan^{-1}(-4/(-3)) - \pi = -2.214 \\ z = -5 \end{cases}$$

Sylindrisk: $[r, \theta, z] = [5, -2.214, -5]$

$$\begin{cases} R = \sqrt{9+16+25} = 5\sqrt{2} \\ \phi = \cos^{-1}(-5/5\sqrt{2}) = 3\pi/4 \\ \theta = -2.214 \end{cases}$$

Sfærisk: $[R, \phi, \theta] = [5\sqrt{2}, 3\pi/4, -2.214]$

ii) $l: \vec{r}_1 = \vec{i} - \vec{j} + 2\vec{k}, \quad \vec{v}_1 = -2\vec{i} + \vec{j} + \vec{k}$

$m: \vec{r}_2 = 4\vec{i} + 5\vec{j}, \quad \vec{v}_2 = 5\vec{j} - 2\vec{k}$

$$\vec{r}_2 - \vec{r}_1 = (4-1)\vec{i} + (5-(-1))\vec{j} + (0-2)\vec{k} = 3\vec{i} + 6\vec{j} - 2\vec{k}$$

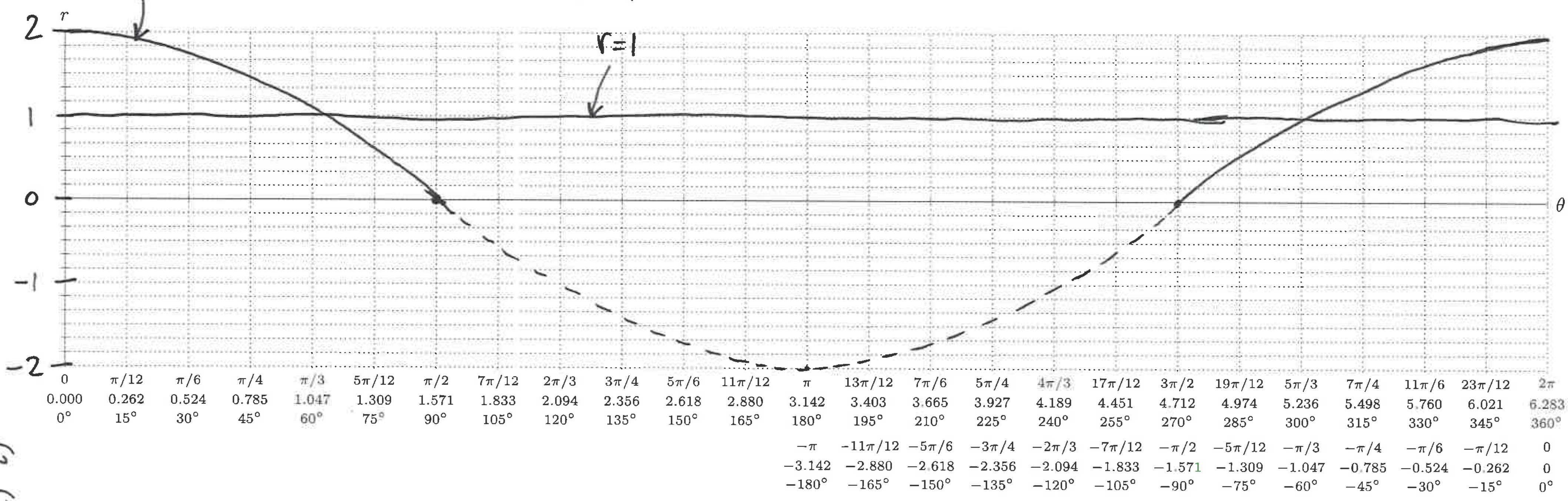
$$\vec{v}_1 \times \vec{v}_2 = \begin{bmatrix} 1 & & 1 & & -2 & & 1 \\ & \times & & \times & & \times & \\ 5 & & -2 & & 0 & & 5 \end{bmatrix} = -7\vec{i} - 4\vec{j} - 10\vec{k}$$

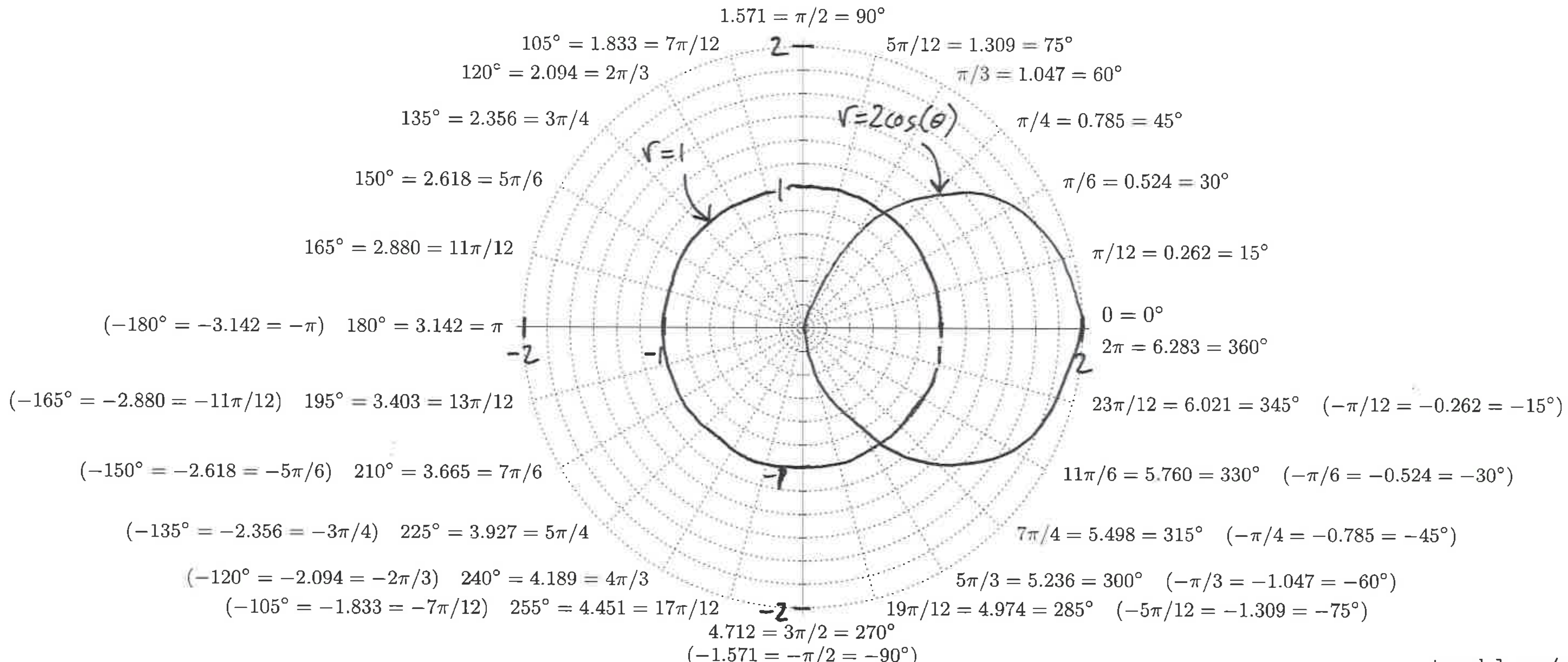
$$s = \frac{|(\vec{r}_2 - \vec{r}_1) \cdot (\vec{v}_1 \times \vec{v}_2)|}{|\vec{v}_1 \times \vec{v}_2|} = \frac{|(3\vec{i} + 6\vec{j} - 2\vec{k}) \cdot (-7\vec{i} - 4\vec{j} - 10\vec{k})|}{|-7\vec{i} - 4\vec{j} - 10\vec{k}|}$$

$$s = \frac{|-21 - 24 + 20|}{\sqrt{7^2 + 4^2 + 10^2}} = \frac{25}{\sqrt{165}} \approx 1.946$$

Løsningsforslag 3 c) i)

$r = 2\cos(\theta) = 0 \Rightarrow \cos(\theta) = 0 \Rightarrow \theta = \{\pi/2, 3\pi/2\}$





trondal.com/polar.pdf

c) i) Se polar-ark.

ii) $r = f(\theta) = 2\cos(\theta)$ og $r = g(\theta) = 1$, Skjæringspunkter:

1. $f(\theta) = 0$

$2\cos(\theta) = 0$

$\theta = \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

$g(\theta) = 0$

$1 = 0$

Ingen løsning.

$\Rightarrow$ ikke skjæring i origo.

2. $f(\theta) = g(\theta)$

$2\cos(\theta) = 1$

$\cos(\theta) = \frac{1}{2}$

$\theta = \left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

$\Rightarrow$ Skjæringspunkter:

$\left[1, \frac{\pi}{3}\right]$ og $\left[1, \frac{5\pi}{3}\right]$

3. $f(\theta + (2k+1)\pi) = -g(\theta)$

$2\cos\left(\theta + 2k\pi + \pi\right) = -1$

$2\cos(\theta + \pi) = -1$

$\cos(\theta + \pi) = -\frac{1}{2}$

$\theta + \pi = \left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

$\theta = \left\{-\frac{\pi}{3}, \frac{\pi}{3}\right\}$

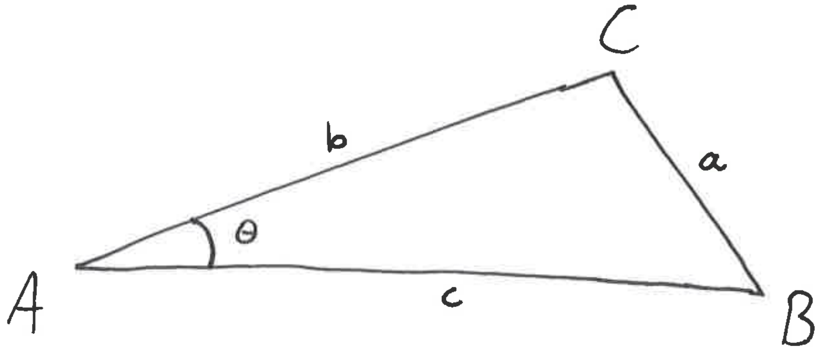
Samme som fra 2.

Skjæring i $\left[1, -\frac{\pi}{3}\right] = \left[1, -\frac{\pi}{3} + 2\pi\right] = \left[1, \frac{5\pi}{3}\right]$ (samme som fra 2.)

d) i) $|\vec{AB}| = \sqrt{(1-2)^2+(1-(-4))^2+(5-2)^2} = \sqrt{35} = c$

$|\vec{AC}| = \sqrt{(0-2)^2+(0-(-4))^2+(3-2)^2} = \sqrt{21} = b$

$|\vec{BC}| = \sqrt{(0-1)^2+(0-1)^2+(3-5)^2} = \sqrt{6} = a$



$$a^2 = b^2 + c^2 - 2bc \cdot \cos(\theta)$$

$$\theta = \cos^{-1}\left(\frac{b^2+c^2-a^2}{2bc}\right) = \cos^{-1}\left(\frac{21+35-6}{2\cdot\sqrt{21}\cdot\sqrt{35}}\right) =$$

$$\theta = \cos^{-1}\left(\frac{5\sqrt{15}}{21}\right) = 0.397 = 22.76^\circ$$

ii) Se neste ark.

d) ii)

punktene er skjæringen mellom ~~en~~ skallet/veggen til en sylinder med radius 3 ~~som~~ og senter langs z-aksen, og planet $z = 0$. Dette blir en sirkel i xy-planet med senter i origo og radius 3.