(3) a) i) $\vec{r}(t) = -t \cdot \vec{i} + (t+t^2)\vec{j} - 15\vec{k}$

$\vec{v}(t) = \vec{r}\,'(t) = -\vec{i} + (1+2t)\vec{j}$

$v(t) = \sqrt{(-1)^2 + (1+2t)^2} = \sqrt{4t^2+4t+2}$

$\vec{a}(t) = \vec{v}\,'(t) = 2\vec{j}$

a) ii) $x = -t^2 + t - 1$

$x' = -2t + 1 = 0 \Rightarrow t = \frac{1}{2} \Rightarrow$ 1 vertikal tangent

$y = -2t^3 + 8t$

$y' = -6t^2 + 8 = 0$

$t^2 = \frac{8}{6} = \frac{4}{3}$

$t = \pm \frac{2}{\sqrt{3}} \Rightarrow$ 2 horisontale tangenter

3 tangenter til sammen

b) i) $9x^2+54x-4y^2+16y+101=0$

$$9\left(x+\frac{54}{2\cdot 9}\right)^2-\frac{54^2}{4\cdot 9}-4\left(y+\frac{16}{2\cdot(-4)}\right)^2-\frac{16^2}{4\cdot(-4)}+101=0$$

$$9(x+3)^2-4(y-2)^2-81+16+101=0$$

$$\frac{9(x+3)^2}{9\cdot 4}-\frac{4(y-2)^2}{9\cdot 4}+\frac{36}{9\cdot 4}=0$$

$$\frac{(x+3)^2}{2^2}-\frac{(y-2)^2}{3^2}=-1$$

$$\frac{(y-2)^2}{3^2}-\frac{(x+3)^2}{2^2}=1$$

Hyperbel med brennpunkter $\parallel$ med y-aksen.

Senter: $(-3,2)$

Senter-toppunkt: 3

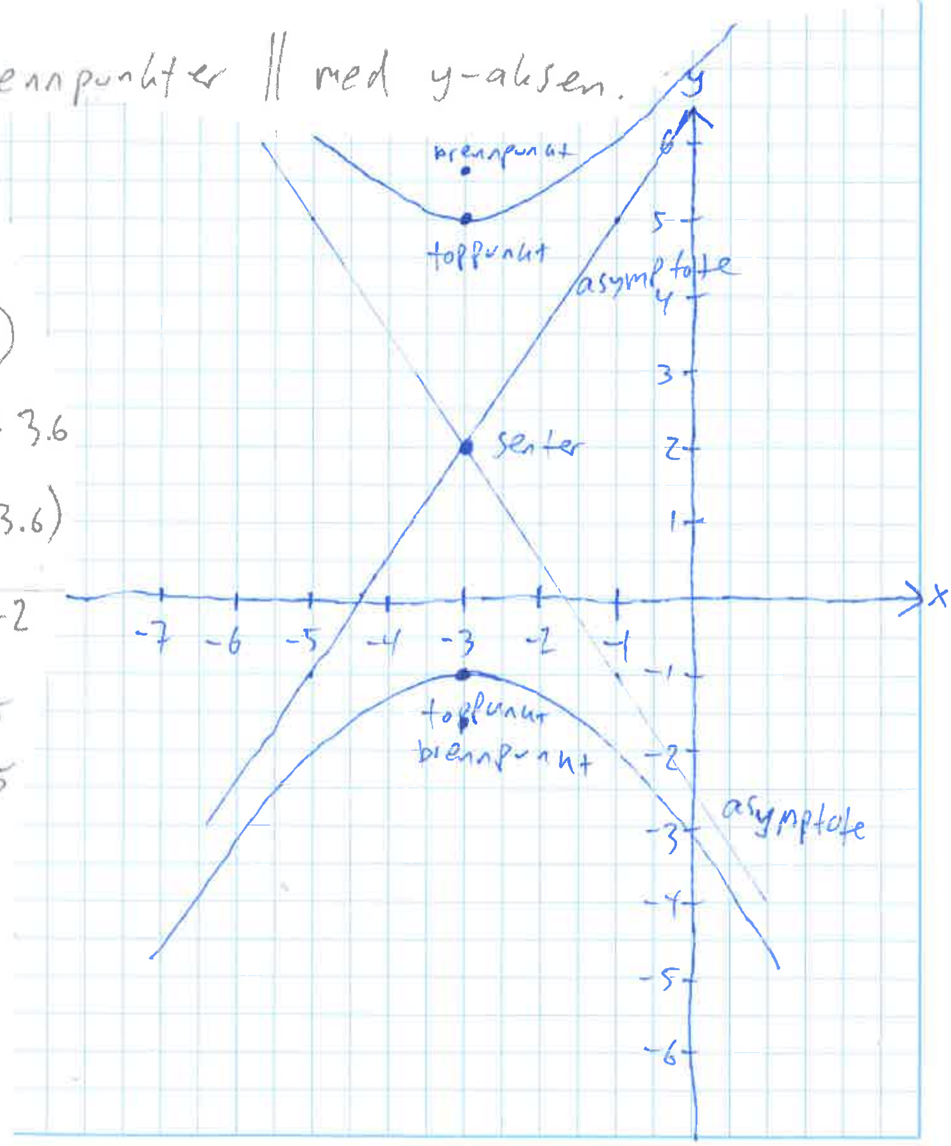
Toppunkt: $(-3, 2\pm 3)$

Senter-brennpunkt: $\sqrt{13}\approx 3.6$

Brennpunkt: $(-3, 2\pm 3.6)$

Asymptoter: $y=\frac{\pm 3}{2}(x+3)+2$

$$y=\begin{cases}1.5x+6.5\\ -1.5x-2.5\end{cases}$$



b) ii) $r = \dfrac{4}{2\cdot\cos(\theta) - 3\cdot\sin(\theta)}$

$$2r\cdot\cos(\theta) - 3r\cdot\sin(\theta) = 4$$

$$2x - 3y = 4$$

Likningen beskriver en rett linje

c) i) $\vec{u} = (-1)\cdot\vec{i} + 0\cdot\vec{j} + 1\cdot\vec{k}$

$\vec{v} = 0\cdot\vec{i} + 4\cdot\vec{j} + (-1)\cdot\vec{k}$

$$\vec{w} = \vec{u}\times\vec{v} = \begin{bmatrix} 0 & & 1 & & -1 & & 0 \\ & \times & & \times & & \times & \\ 4 & & -1 & & 0 & & 4 \end{bmatrix} = -4\vec{i} - \vec{j} - 4\vec{k}$$

$$|\vec{w}| = |\vec{u}\times\vec{v}| = \sqrt{4^2+1^2+4^2} = \sqrt{33}$$

$$\frac{\vec{w}}{|\vec{w}|} = -\frac{4}{\sqrt{33}}\vec{i} - \frac{1}{\sqrt{33}}\vec{j} - \frac{4}{\sqrt{33}}\vec{k}$$

f) ii) $A = (0,0,0) \quad B = (-1,0,1) \quad C = (0,4,-1)$

$\vec{AB} = -\vec{i} + \vec{k} = \vec{u}$ fra oppgave e

$\vec{AC} = 4\vec{j} - \vec{k} = \vec{v}$ fra oppgave e

$$\text{Areal} = \frac{1}{2} \cdot |\vec{AB} \times \vec{AC}| = \frac{1}{2} \cdot |\vec{u} \times \vec{v}| = \frac{1}{2} |\vec{w}| = \frac{\sqrt{33}}{2}$$

d VIII) i) $\displaystyle\int_0^2 \sqrt{(x')^2 + (y')^2}\,dt = \int_0^2 \sqrt{(\cos(t^2)')^2 + (\sin(t^2)')^2}\,dt$

$$= \int_0^2 \sqrt{(-\sin(t^2)\cdot 2t)^2 + (\cos(t^2)\cdot 2t)^2}\,dt$$

$$= \int_0^2 \sqrt{4t^2\cdot(\sin(t^2)^2 + \cos(t^2)^2)}\,dt$$

$$= \int_0^2 2t\,dt = \left[t^2\right]_0^2 = 4$$

d) ii) Begge grafene har $r>0$ for alle verdier av θ, og har da kun evt. skjæringspunkter når

$$3+2\cdot\sin(\theta)=2$$

$$2\cdot\sin(\theta)=-1$$

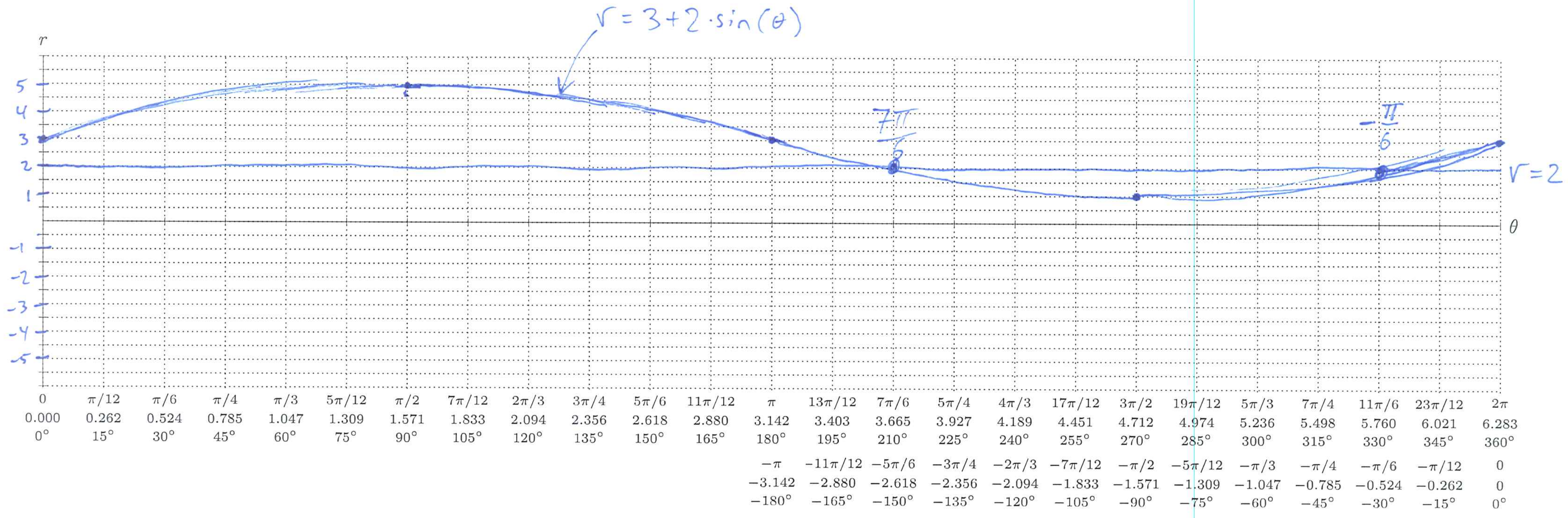
$$\sin(\theta)=-\frac{1}{2}$$

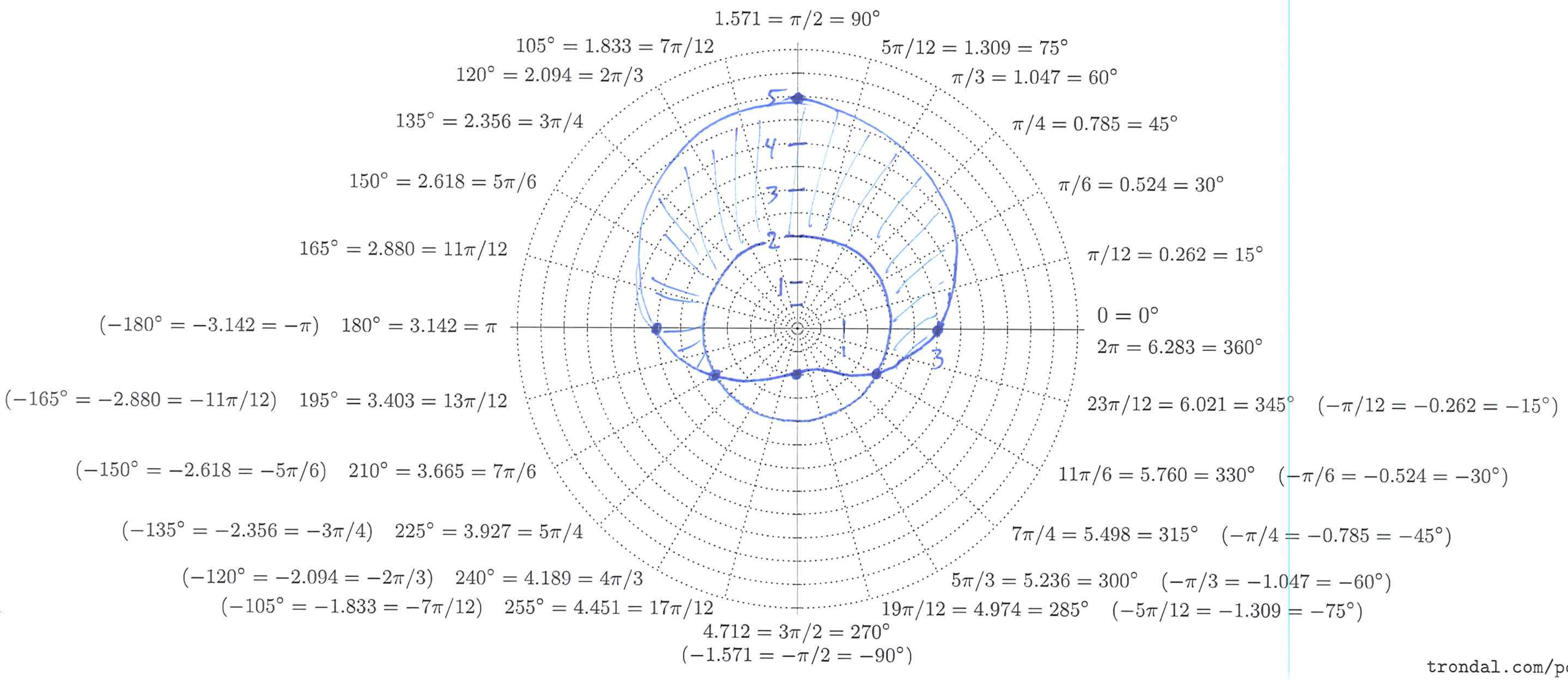
$$\theta=\left\{-\frac{\pi}{6},\ \frac{7\pi}{6}\right\}$$

$$\text{Areal}=\frac{1}{2}\int_{-\frac{\pi}{6}}^{\frac{7\pi}{6}}\left(\left(3+2\sin(\theta)\right)^2-2^2\right)d\theta=\frac{28\pi+33\sqrt{3}}{6}$$

$$\approx 24.187 \quad \text{(kalkulator)}$$

d) ii)





trondal.com/polar.pdf