11.1.1 $\vec{r}(t) = \vec{i} + t\vec{j}$

$\vec{r}'(t) = \vec{v}(t) = \vec{j}$

$|\vec{v}(t)| = v(t) = \sqrt{1^2} = 1$

$\vec{v}'(t) = \vec{a}(t) = \vec{0}$

Bevegelsen er i en rett linje, gjennom punktet $(1,0,0)$, parallell med y-aksen, mot positiv y-akse, har jevn fart og ingen akselerasjon.

11.1.2 $\vec{r}(t) = t^2\vec{i} + \vec{k}$

$\vec{r}'(t) = \vec{v}(t) = 2t\vec{i}$

$|\vec{v}(t)| = v(t) = \sqrt{(2t)^2} = 2t$

$\vec{v}'(t) = \vec{a}(t) = 2\vec{i}$

Partikkelens bane er en rett linje; Det er kun x-koordinaten som endrer seg. Ved

$t = -\infty$, $t = 0$, $t = \infty$

er x-komponenten til partikkelen

$x = \infty$, $x = 0$, $x = \infty$.

y og z komponentene er konstante på hhv.

$y = 0$ og $z = 1$.

11.1.3 $\vec{r}(t) = t^2\vec{j} + t\vec{k}$

$\vec{r}'(t) = \vec{v}(t) = 2t\vec{j} + \vec{k}$

$|\vec{v}(t)| = v(t) = \sqrt{(2t)^2 + 1^2} = \sqrt{4t^2 + 1}$

$\vec{v}'(t) = \vec{a}(t) = 2\vec{j}$

Bane: $y = t^2$, $z = t$

$$\Downarrow$$

$$y = z^2$$

Dvs en parabel i yz-planet.

11.1.4 $\vec{r}(t) = \vec{i} + t\vec{j} + t\vec{k}$

$\vec{r}'(t) = \vec{v}(t) = \vec{j} + \vec{k}$

$|\vec{v}(t)| = v(t) = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\vec{v}'(t) = \vec{a}(t) = \vec{0}$

$$\begin{matrix} x = 1 \\ y = t \\ z = t \end{matrix} \Rightarrow \begin{matrix} x = 1 \\ y = z \end{matrix}$$

$$\Downarrow$$

Dvs en rett linje $y = z$ med $x = 1$ (konstant)

11.1.5 $\vec{r}(t) = t^2\vec{i} - t^2\vec{j} + \vec{k}$

$\vec{r}'(t) = \vec{v}(t) = 2t\vec{i} - 2t\vec{j}$

$|\vec{v}(t)| = v(t) = \sqrt{(2t)^2 + (-2t)^2} = 2\sqrt{2}\,t$

$\vec{v}'(t) = \vec{a}(t) = 2\vec{i} - 2\vec{j}$

$$\begin{matrix} x = t^2 \\ y = t^2 \\ z = 1 \end{matrix} \Rightarrow \begin{matrix} x = y \\ z = 1 \end{matrix}$$

$$\Downarrow$$

Dvs en rett linje $y = x$ med $z = 1$ (konstant)

11.1.6 $\vec{r}(t) = t\vec{i} + t^2\vec{j} + t^2\vec{k}$

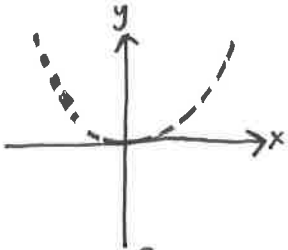
$$\vec{r}'(t) = \vec{v}(t) = \vec{i} + 2t\vec{j} + 2t\vec{k}$$

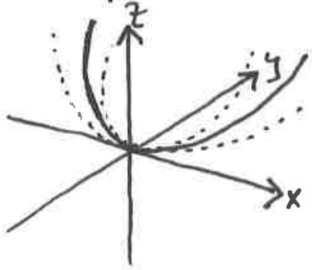
$$|\vec{v}(t)| = v(t) = \sqrt{1^2 + (2t)^2 + (2t)^2} = \sqrt{8t^2+1}$$

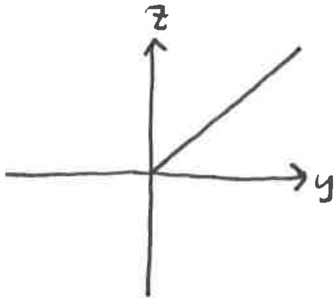
$$\vec{v}'(t) = \vec{a}(t) = 2\vec{j} + 2\vec{k}$$

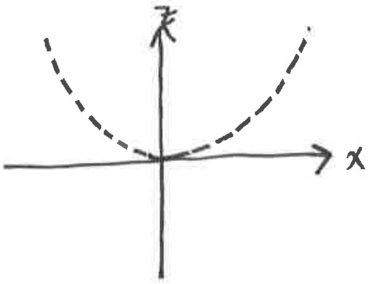
$$\begin{aligned} x &= t \\ y &= t^2 \\ z &= t^2 \end{aligned} \Rightarrow \begin{aligned} y &= x^2 \\ z &= x^2 \end{aligned} \Rightarrow y = z$$

Dvs en slags dobbel parabel-bevegelse.









11.1.7 $\vec{r}(t) = a \cdot \cos t \cdot \vec{i} + a \cdot \sin t \cdot \vec{j} + c \cdot t \cdot \vec{k}$

$$\vec{r}'(t) = \vec{v}(t) = -a\sin t \cdot \vec{i} + a \cdot \cos t \cdot \vec{j} + c\vec{k}$$

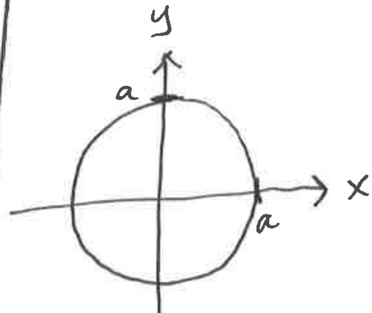
$$|\vec{v}(t)| = v(t) = \sqrt{a^2\sin^2 t + a^2\cos^2 t + c^2} = \sqrt{a^2 + c^2}$$

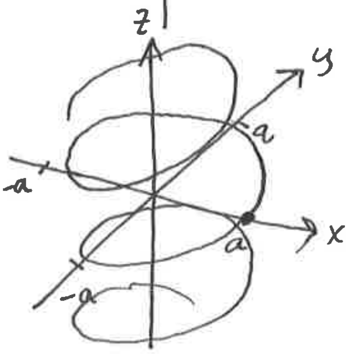
$$\vec{v}'(t) = \vec{a}(t) = -a \cdot \cos t\vec{i} - a \cdot \sin t \cdot \vec{j}$$

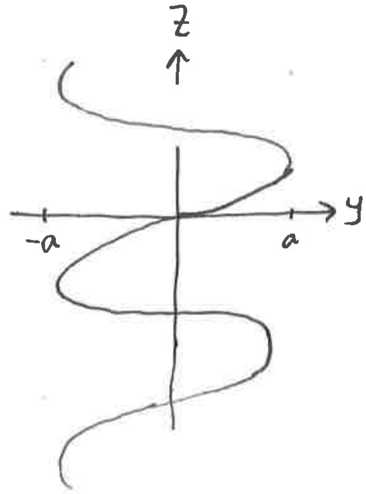
$$\begin{aligned} x &= a \cdot \cos t \\ y &= a \cdot \sin t \\ z &= c \cdot t \end{aligned} \Rightarrow \begin{aligned} x^2 &= a^2 \cdot \cos^2 t \\ y^2 &= a^2 \cdot \sin^2 t \\ z &= c \cdot t \end{aligned}$$

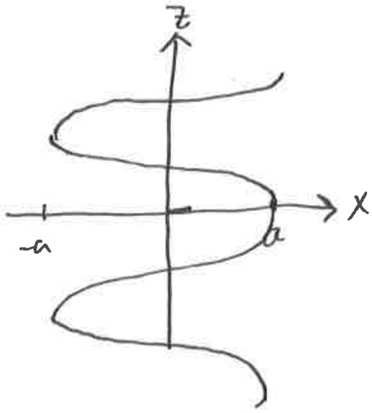
$$\Rightarrow x^2 + y^2 = a^2(\cos^2 t + \sin^2 t) = a^2$$
$$z = c \cdot t$$

Aha! Dette må være en spiral:









$\underline{11.1.8}$ $\vec{r}(t) = a\cdot\cos(\omega t)\vec{i} + b\vec{j} + a\cdot\sin(\omega t)\vec{k}$

$\vec{r}'(t) = \vec{v}(t) = -a\cdot\omega\cdot\sin(\omega t)\vec{i} + a\cdot\omega\cdot\cos(\omega t)\vec{k}$

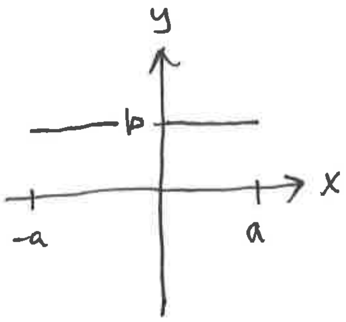
$|\vec{v}(t)| = v(t) = \sqrt{a^2\omega^2\sin^2(\omega t) + a^2\omega^2\cos^2(\omega t)} = a\omega$

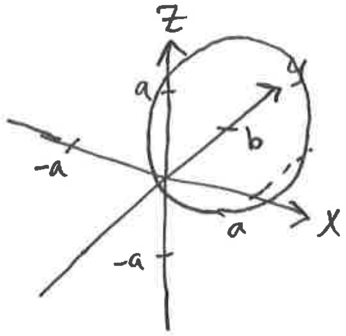
$\vec{v}'(t) = \vec{a}(t) = -a\cdot\omega^2\cdot\cos(\omega t)\vec{i} - a\cdot\omega^2\cdot\sin(\omega t)\vec{k}$

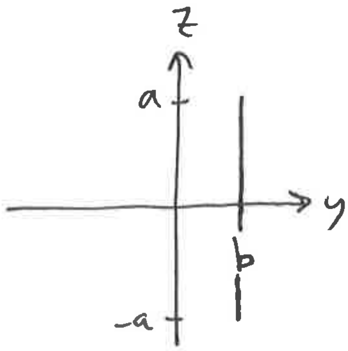
$$x = a\cdot\cos(\omega t)$$
$$y = b$$
$$z = a\cdot\sin(\omega t)$$

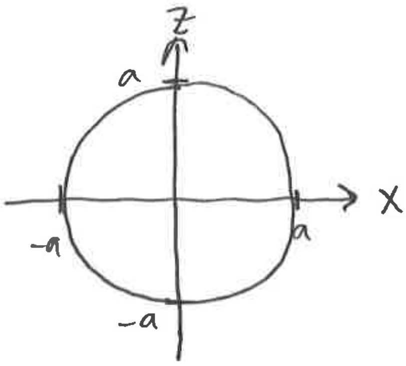
$\Rightarrow$ $x^2 + z^2 = a^2$, $y = b$ (se løsning for 10.1.7)

En sirkel:









11.1.11. $\vec{r}(t) = a \cdot e^t \cdot \vec{i} + b \cdot e^t \cdot \vec{j} + c \cdot e^t \cdot \vec{k}$

$\vec{r}'(t) = \vec{v}(t) = a \cdot e^t \cdot \vec{i} + b e^t \vec{j} + c e^t \cdot \vec{k}$

$|\vec{v}(t)| = v(t) = \sqrt{a^2 e^{2t} + b^2 e^{2t} + c^2 e^{2t}} = e^t \cdot \sqrt{a^2 + b^2 + c^2}$

$\vec{v}'(t) = \vec{a}(t) = a \cdot e^t \cdot \vec{i} + b e^t \vec{j} + c e^t \vec{k}$

$x = a \cdot e^t$

$y = b \cdot e^t \quad \Rightarrow \quad e^t = \frac{x}{a} = \frac{y}{b} = \frac{z}{c}$

$z = c \cdot e^t$

Som er en rett linje jfr. kap. 10.4.