8.3.1

$x = t^2+1$, $y = 2t-4$

$x' = 0$

$2t = 0$

$t = 0$

$\Downarrow$

Vertikal tangent når $t=0$, dvs i punktet

$x(0) = 0^2+1 = 1$

$y(0) = 2\cdot 0 - 4 = -4$

$(1,-4)$.

$y' = 0$

$2 = 0$

$\Downarrow$

Ingen horisontal tangent

8.3.2

$x = t^2-2t$, $x'=0 \Rightarrow t=1$

$y = t^2+2t$, $y'=0 \Rightarrow t=-1$

$x(1) = -1$, $y(1) = 3$

$x(-1) = 3$, $y(-1) = -1$

$\Downarrow$

Vertikal tangent i $(-1,3)$

Horisontal tangent i $(3,-1)$

8.3.3

$x = t^2-2t$, $x'=0 \Rightarrow t=1$

$y = t^3-12t$, $y'=0 \Rightarrow 3t^2-12=0$

$t^2 = 4$

$t = \pm 2$

$x(1) = -1$, $y(1) = -11$

$x(2) = 0$, $y(2) = -16$

$x(-2) = 8$, $y(-2) = -8+24 = 16$

$\Downarrow$

Vertikal tangent i $(-1,-11)$

Horisontal tangent i $(0,-16)$ og $(8,16)$

8.3.4

$x = t^3-3t$, $x'=0 \Rightarrow 3t^2-3=0$

$t^2 = 1$

$t = \pm 1$

$y = 2t^3+3t^2$, $y'=0 \Rightarrow 6t^2+6t=0$

$t^2+t=0$

$t(t+1)=0$

$t=0$ og $t=-1$

$x(1) = -2$, $y(1) = 5$

$x(0) = 0$, $y(0) = 0$

$\Downarrow$

Vertikal tangent i: $(-2,5)$

Horisontal tangent i: $(0,0)$

NB! $x'=y'=0$ når $t=-1$, og ingen tangent der...

8.3.5

$$x = te^{(-t^2/2)}, \quad y = e^{-t^2}$$

$$x' = (t)' \cdot e^{-t^2/2} + t \cdot \left(e^{-t^2/2}\right)'$$

$$= 1 \cdot e^{-t^2/2} + t \cdot \left(e^{-t^2/2} \cdot (-2t/2)\right)$$

$$= e^{-t^2/2} \cdot (1 - t^2) = 0 \Rightarrow t = \pm 1$$

$$y' = e^{-t^2} \cdot (-2t) = -2te^{-t^2} = 0 \Rightarrow t = 0$$

$$x(1) = e^{-1/2}, \quad y(1) = e^{-1}$$

$$x(-1) = -e^{-1/2}, \quad y(-1) = e^{-1}$$

$$x(0) = 0, \quad y(0) = 1$$

Vertikale tangenter: $(\pm e^{-1/2}, e^{-1})$

Horisontal tangent: $(0, 1)$

8.3.8

$$x = \frac{3t}{1+t^3}, \quad y = \frac{3t^2}{1+t^3}$$

$$x' = \frac{3(1+t^3) - 3t(3t^2)}{(1+t^3)^2} = 0$$

[Hvis $t = -1$ så finnes ikke x', så $t = -1$ er ikke noen løsning. Dvs. at vi kan gange med $(1+t^3)^2$:]

$$3 + 3t^3 - 9t^3 = 0$$

$$3 - 6t^3 = 0$$

$$1 - 2t^3 = 0$$

$$t^3 = \frac{1}{2} \Rightarrow t = \sqrt[3]{\frac{1}{2}}$$

$x\left(\sqrt[3]{\frac{1}{2}}\right) = 2 \cdot \sqrt[3]{\frac{1}{2}}$, $y\left(\sqrt[3]{\frac{1}{2}}\right) = 2 \cdot \sqrt[3]{\frac{1}{2}}$, dvs:

Vertikal tangent ved $\left(2 \cdot \sqrt[3]{\frac{1}{2}}, 2 \cdot \sqrt[3]{\frac{1}{2}}\right)$.

$$y' = \frac{6t \cdot (1+t^3) - 3t^2 \cdot (3t^2)}{(1+t^3)^2} = 0$$

[samme situasjon her.]

$$6t + 6t^4 - 9t^4 = 0$$

$$6t - 3t^4 = 0$$

$$t(2 - t^3) = 0$$

$t = 0$: $x(0) = 0$, $y(0) = 0$

$t^3 = 2 \Rightarrow t = \sqrt[3]{2}$: $x(\sqrt[3]{2}) = \sqrt[3]{2}$, $y(\sqrt[3]{2}) = \sqrt[3]{4}$

Horisontale tangenter i $(0, 0)$ og i $(\sqrt[3]{2}, \sqrt[3]{4})$

8.3.9

$$x = t^3 + t, \quad y = 1 - t^3$$

$$x' = 3t^2 + 1, \quad y' = -3t^2$$

$$\left.\frac{y'}{x'}\right|_{t=1} = \frac{-3}{4} = -\frac{3}{4}$$

8.3.10

$$x = t^4 - t^2, \quad y = t^3 + 2t$$

$$x' = 4t^3 - 2t, \quad y' = 3t^2 + 2$$

$$\left.\frac{y'}{x'}\right|_{t=-1} = \frac{5}{-2} = -\frac{5}{2}$$

8.3.11

$$x = \cos(2t), \quad y = \sin(t)$$

$$x' = -2\sin(2t), \quad y' = \cos(t)$$

$$\left.\frac{y'}{x'}\right|_{t=\frac{\pi}{6}} = \frac{\frac{\sqrt{3}}{2}}{-\sqrt{3}} = -\frac{1}{2}$$

8.3.12

$$x = e^{2t}, \quad y = t \cdot e^{2t}$$

$$x' = 2e^{2t}, \quad y' = e^{2t} + 2te^{2t} = e^{2t}(1+2t)$$

$$\left.\frac{y'}{x'}\right|_{t=-2} = \left.\frac{\cancel{e^{2t}}(1+2t)}{2\cancel{e^{2t}}}\right|_{t=-2} = -\frac{3}{2}$$

8.3.13

$x = t^3 - 2t, \quad y = t + t^3, \quad t = 1$

$x' = 3t^2 - 2, \quad y' = 1 + 3t^2$

$x(1) = -1, \quad y(1) = 2$

$x'(1) = 1, \quad y'(1) = 4$

Tangent:

$$\begin{cases} x = -1 + 1\cdot(t-1) = t-2 \\ y = 2 + 4\cdot(t-1) = 4t-2 \end{cases}$$

8.3.14

$x = t - \cos(t), \quad y = 1 - \sin(t), \quad t = \frac{\pi}{4}$

$x' = 1 + \sin(t), \quad y = -\cos(t)$

$x\left(\frac{\pi}{4}\right) = \frac{\pi}{4} - \cos\left(\frac{\pi}{4}\right) = \frac{\pi}{4} - \frac{\sqrt{2}}{2}$

$y\left(\frac{\pi}{4}\right) = 1 - \sin\left(\frac{\pi}{4}\right) = 1 - \frac{\sqrt{2}}{2}$

$x'\left(\frac{\pi}{4}\right) = 1 + \sin\left(\frac{\pi}{4}\right) = 1 + \frac{\sqrt{2}}{2}$

$y'\left(\frac{\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

Tangent:

$$\begin{cases} x = \frac{\pi}{4} - \frac{\sqrt{2}}{2} + \left(1 + \frac{\sqrt{2}}{2}\right)\left(t - \frac{\pi}{4}\right) \\ y = 1 - \frac{\sqrt{2}}{2} + \left(-\frac{\sqrt{2}}{2}\right)\left(t - \frac{\pi}{4}\right) \end{cases}$$

8.3.21

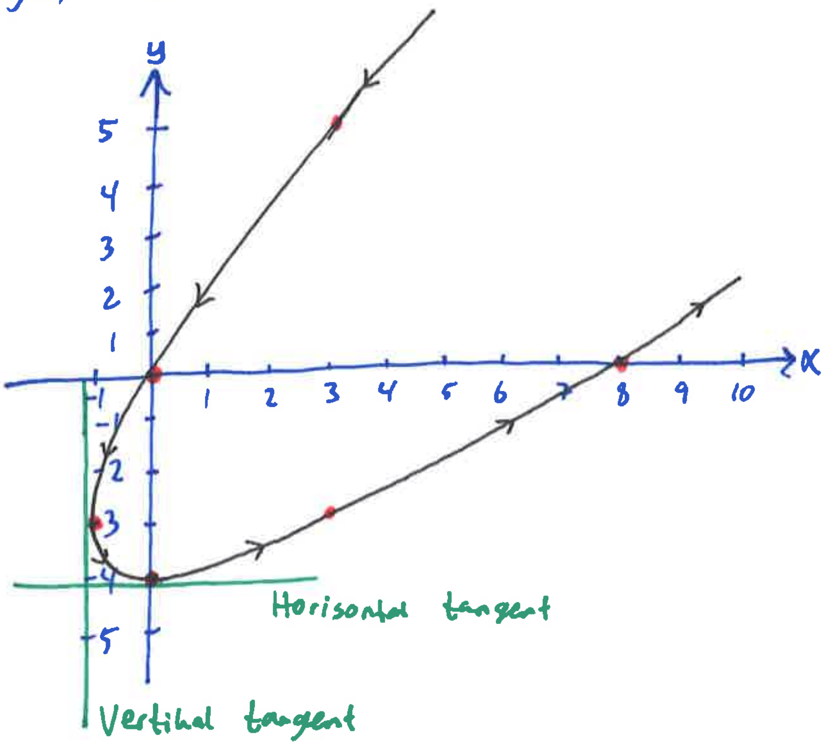
$x = t^2 - 2t, \quad y = t^2 - 4t$

$x' = 2t - 2, \quad y' = 2t - 4$

$x' = 0 \Rightarrow t = 1, \quad y' = 0 \Rightarrow t = 2$

Vertikal ↑ Horisontal ↑

t	-1		0		1		2		3		4
$x'(t)$	$-$	$-$	$-$	$-$	0	$+$	$+$	$+$	$+$	$+$	$+$
$y'(t)$	$-$	$-$	$-$	$-$	$-$	$-$	0	$+$	$+$	$+$	$+$
x	←	←	←	←	·	→	→	→	→	→	→
y	↓	↓	↓	↓	↓	↓	·	↑	↑	↑	↑
Curve	↙	↙	↙	↙	↓	↘	→	↗	↗	↗	↗
$x(t)$	3		0		-1		0		3		8
$y(t)$	5		0		-3		-4		-3		0



8.3.23

$x = t^3 - 3t, \quad y = \frac{2}{1+t^2}$

$x' = 3t^2 - 3 = 0 \Rightarrow t = \pm 1$

$y' = \frac{-2 \cdot (2t)}{(1+t^2)^2}$

$= \frac{-4t}{(t^2+1)^2} = 0$

$\Rightarrow \quad t = 0$

$x(1) = -2, \; y(1) = 1$

$x(-1) = 2, \; y(-1) = 1$

Dvs, vertikale tangenter ved $(\pm 2, 1)$

$x(0) = 0, \; y(0) = 2$

Dvs, horisontal tangent ved $(0, 2)$

t	-3	-2	-1	0	1	2	3
$x(t)$	-18	-2	2	0	-2	2	18
$y(t)$	0.2	0.4	1	2	1	0.4	0.2

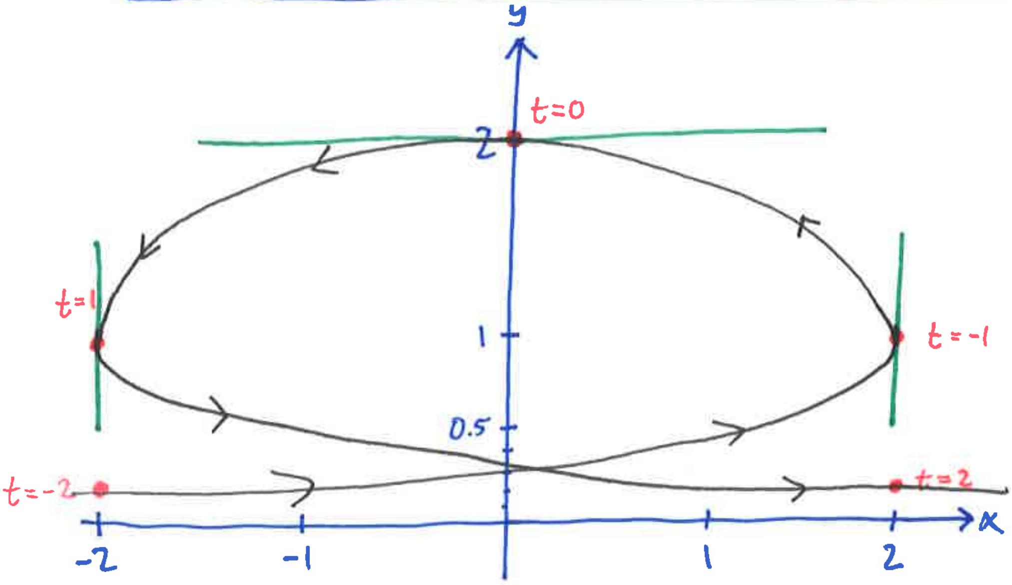
$x'(t)$: + + + + 0 − − − 0 + + + +

$y'(t)$: + + + + + + 0 − − − − − −

x: → → → → · ← ← ← · → → → →

y: ↑ ↑ ↑ ↑ ↑ ↑ · ↓ ↓ ↓ ↓ ↓ ↓

curve: ↗ ↗ ↗ ↗ ↑ ↖ ← ↙ ↓ ↘ ↘ ↘ ↘



8.3.24

$x = t^3 - 3t - 2, \quad y = t^2 - t - 2$

$x' = 3t^2 - 3 = 0 \Rightarrow t = \pm 1$

$y' = 2t - 1 = 0 \Rightarrow t = \frac{1}{2}$

$x(1) = -4, \quad y(1) = -2$

$x(-1) = 0, \quad y(-1) = 0$

$x(\frac{1}{2}) = -3.375, \quad y(\frac{1}{2}) = -2.25$

Vertikale tangenter ved:

$(-4, -2)$ og $(0, 0)$

Horisontal tangent ved:

$(-3.375, -2.25)$

t	-2.5	-2		-1		0		1		2	2.5
$x(t)$	-10.1	-4		0		-2		-4		0	6.1
$y(t)$	6.75	4		0		-2		-2		0	1.75
$x'(t)$	$+$	$+$	$+$	0	$-$	$-$	$-$	0	$+$	$+$	$+$
$y'(t)$	$-$	$-$	$-$	$-$	$-$	$-$	0	$+$	$+$	$+$	$+$
x	→	→	→	·	←	←	←	·	→	→	→
y	↓	↓	↓	↓	↓	↓	·	↑	↑	↑	↑
Curve	↘	↘	↘	↓	↙	↙	←	↑	↗	↗	↗