

8.5.25

$$r = f(\theta) = \sqrt{3} \cos \theta$$

$$r = g(\theta) = \sin \theta$$

1) $f(\theta) = 0$

$$\sqrt{3} \cos \theta = 0$$

$$\cos \theta = 0$$

$$\theta = \left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\} \text{ ok.}$$

$$g(\theta) = 0$$

$$\sin \theta = 0$$

$$\theta = \{0, \pi\} \text{ ok.}$$

\Rightarrow Skjæring i origo; $(0,0)$

2) $f(\theta) = g(\theta)$

$$\sqrt{3} \cos \theta = \sin \theta$$

$$\tan \theta = \sqrt{3}$$

$$\theta = \left\{ \frac{\pi}{3}, \frac{4\pi}{3} \right\}$$

$$g\left(\frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$g\left(\frac{4\pi}{3}\right) = \sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

\Rightarrow Skjæring i $\left(\frac{\sqrt{3}}{2}, \frac{\pi}{3}\right)$

og $\left(-\frac{\sqrt{3}}{2}, \frac{4\pi}{3}\right)$

$$= \left[\frac{\sqrt{3}}{3}, \frac{4\pi}{3} - \pi\right]$$

$$= \left[\frac{\sqrt{3}}{3}, \frac{\pi}{3}\right]$$

dvs, samme punkt

3) $f(\theta + (2k+1)\pi) = -g(\theta)$

$$\sqrt{3} \cos(\theta + (2k+1)\pi) = -\sin \theta$$

$$\sqrt{3} \cos(\theta + 2k\pi + \pi) = -\sin \theta$$

$$\sqrt{3} \cos(\theta + \pi) = -\sin \theta$$

$$\sqrt{3} (-\cos(\theta)) = -\sin \theta$$

$$\sqrt{3} = \tan \theta$$

$$\theta = \left\{ \frac{\pi}{3}, \frac{4\pi}{3} \right\}$$

dvs samme løsninger som fra 2)

8.5.27

$$r = f(\theta) = 1 + \cos(\theta)$$

$$r = g(\theta) = 3 \cos(\theta)$$

1) $f(\theta) = 0$

$$1 + \cos(\theta) = 0$$

$$\cos(\theta) = -1$$

$$\theta = \pi \quad \text{ok.}$$

$$g(\theta) = 0$$

$$3 \cos(\theta) = 0$$

$$\cos(\theta) = 0$$

$$\theta = \left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\} \text{ ok.}$$

⇒ Skjæring i origo: $(0,0)$

2) $f(\theta) = g(\theta)$

$$1 + \cos(\theta) = 3 \cos(\theta)$$

$$-2 \cos(\theta) = -1$$

$$\cos(\theta) = \frac{1}{2}$$

$$\theta = \left\{ \frac{\pi}{3}, \frac{5\pi}{3} \right\}$$

$$g\left(\frac{\pi}{3}\right) = 3 \cdot \cos\left(\frac{\pi}{3}\right) = \frac{3}{2}$$

$$g\left(\frac{5\pi}{3}\right) = 3 \cdot \cos\left(\frac{5\pi}{3}\right) = \frac{3}{2}$$

⇒ Skjæring i $\left(\frac{3}{2}, \frac{\pi}{3}\right)$

og $\left(\frac{3}{2}, \frac{5\pi}{3}\right)$

3) $f(\theta + (2u+1)\pi) = -g(\theta)$

$$1 + \cos(\theta + (2u+1)\pi) = -3 \cos(\theta)$$

$$1 + \cos(\theta + 2k\pi + \pi) = -3 \cos(\theta)$$

$$1 + \cos(\theta + \pi) = -3 \cos(\theta)$$

$$1 - \cos(\theta) = -3 \cos(\theta)$$

$$2 \cos(\theta) = -1$$

$$\cos(\theta) = -\frac{1}{2}$$

$$\theta = \left\{ \frac{2\pi}{3}, \frac{4\pi}{3} \right\}$$

$$g\left(\frac{2\pi}{3}\right) = -\frac{3}{2}$$

$$g\left(\frac{4\pi}{3}\right) = -\frac{3}{2}$$

⇒ Skjæring i

$$\left[-\frac{3}{2}, \frac{2\pi}{3}\right] = \left[\frac{3}{2}, \frac{2\pi}{3} + \pi\right] = \left[\frac{3}{2}, \frac{5\pi}{3}\right]$$

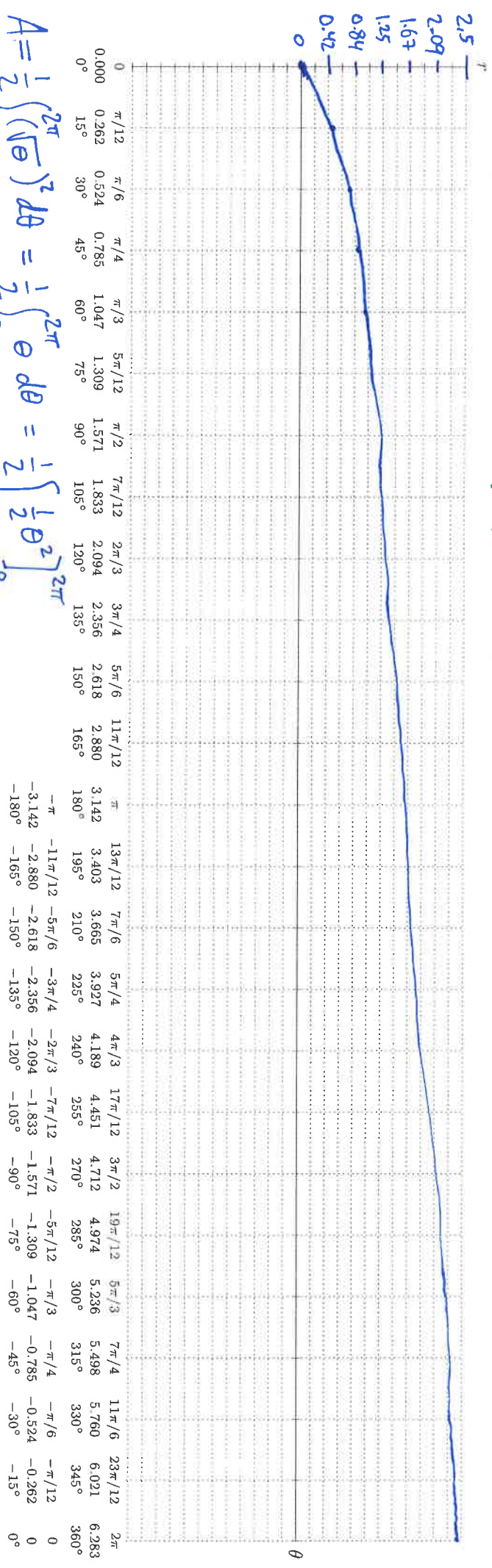
og

$$\left[-\frac{3}{2}, \frac{4\pi}{3}\right] = \left[\frac{3}{2}, \frac{4\pi}{3} - \pi\right] = \left[\frac{3}{2}, \frac{\pi}{3}\right]$$

dvs. samme løsninger som fra 2)

$$r = \sqrt{\theta}$$

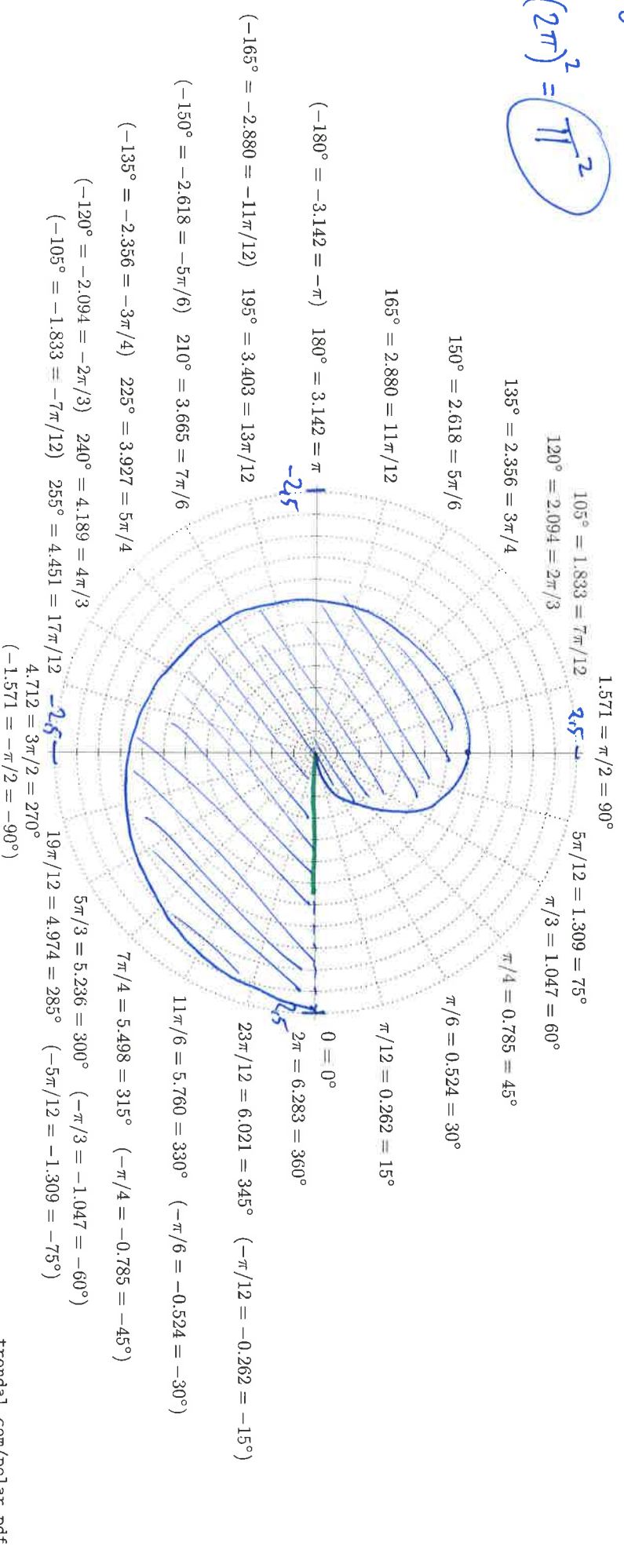
$$r = \sqrt{\theta} = 0 \Rightarrow \theta = 0$$



$$A = \frac{1}{2} \int_0^{2\pi} (\sqrt{\theta})^2 d\theta = \frac{1}{2} \int_0^{2\pi} \theta d\theta = \frac{1}{2} \left[\frac{1}{2} \theta^2 \right]_0^{2\pi}$$

$$= \frac{1}{4} \cdot (2\pi)^2 = \pi^2$$

1.9.8



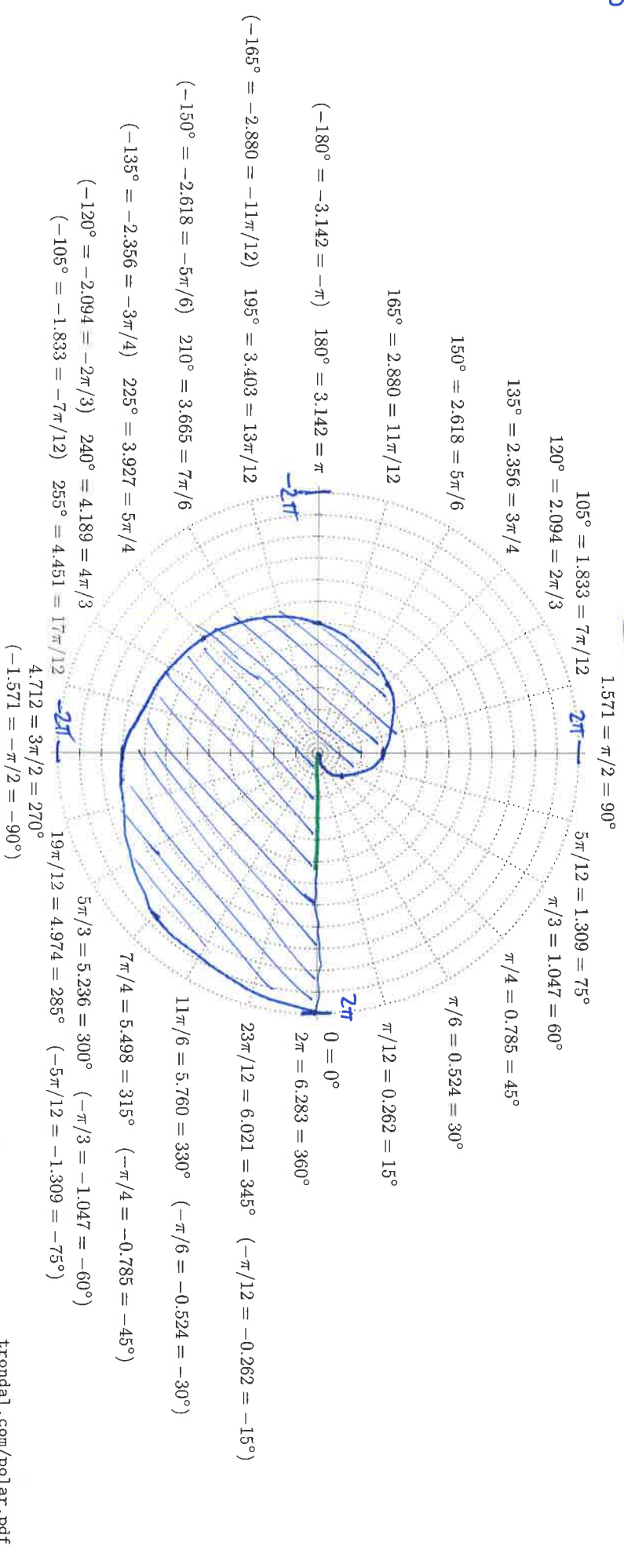
$$r = \theta$$

$$r = \theta = 0 \Rightarrow \theta = 0$$



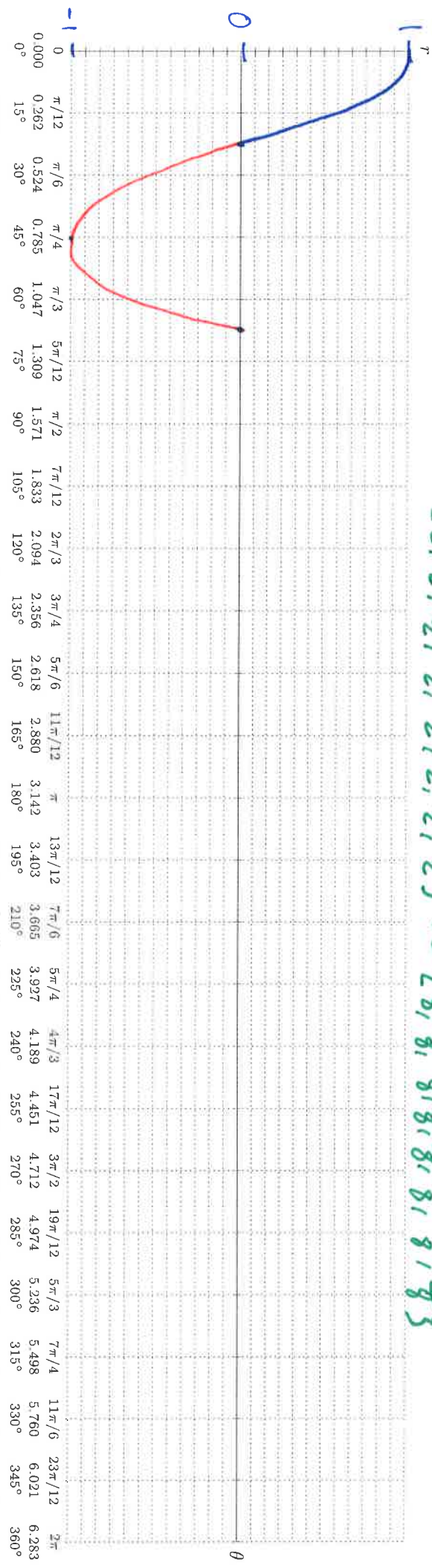
$$A = \frac{1}{2} \int_0^{2\pi} \theta^2 d\theta = \frac{1}{2} \left[\frac{1}{3} \theta^3 \right]_0^{2\pi} = \frac{1}{6} \cdot (2\pi)^3 = \frac{8}{6} \pi^3 = \frac{4}{3} \pi^3$$

8.6.2



- $150^\circ = 2.618 = 5\pi/6$
- $165^\circ = 2.880 = 11\pi/12$
- $180^\circ = 3.142 = \pi$
- $195^\circ = 3.403 = 13\pi/12$
- $210^\circ = 3.665 = 7\pi/6$
- $225^\circ = 3.927 = 5\pi/4$
- $240^\circ = 4.189 = 4\pi/3$
- $255^\circ = 4.451 = 17\pi/12$
- $270^\circ = 4.712 = 3\pi/2 = 270^\circ$
- $285^\circ = 4.974 = 19\pi/12$
- $300^\circ = 5.236 = 5\pi/3$
- $315^\circ = 5.498 = 7\pi/4$
- $330^\circ = 5.760 = 11\pi/6$
- $345^\circ = 6.021 = 23\pi/12$
- $360^\circ = 6.283 = 2\pi$
- $135^\circ = 2.356 = 3\pi/4$
- $120^\circ = 2.094 = 2\pi/3$
- $105^\circ = 1.833 = 7\pi/12$
- $90^\circ = 1.571 = \pi/2 = 90^\circ$
- $75^\circ = 1.309 = 5\pi/12$
- $60^\circ = 1.047 = \pi/3$
- $45^\circ = 0.785 = \pi/4$
- $30^\circ = 0.524 = \pi/6$
- $15^\circ = 0.262 = \pi/12$
- $0^\circ = 0 = 0^\circ$
- $-15^\circ = -0.262 = -\pi/12$
- $-30^\circ = -0.524 = -\pi/6$
- $-45^\circ = -0.785 = -\pi/4$
- $-60^\circ = -1.047 = -\pi/3$
- $-75^\circ = -1.309 = -5\pi/12$
- $-90^\circ = -1.571 = -\pi/2 = -90^\circ$
- $-105^\circ = -1.833 = -7\pi/12$
- $-120^\circ = -2.094 = -2\pi/3$
- $-135^\circ = -2.356 = -3\pi/4$
- $-150^\circ = -2.618 = -5\pi/6$
- $-165^\circ = -2.880 = -11\pi/12$
- $-180^\circ = -3.142 = -\pi$
- $-195^\circ = -3.403 = -13\pi/12$
- $-210^\circ = -3.665 = -7\pi/6$
- $-225^\circ = -3.927 = -5\pi/4$
- $-240^\circ = -4.189 = -4\pi/3$
- $-255^\circ = -4.451 = -17\pi/12$
- $-270^\circ = -4.712 = -3\pi/2 = -270^\circ$
- $-285^\circ = -4.974 = -19\pi/12$
- $-300^\circ = -5.236 = -5\pi/3$
- $-315^\circ = -5.498 = -7\pi/4$
- $-330^\circ = -5.760 = -11\pi/6$
- $-345^\circ = -6.021 = -23\pi/12$
- $-360^\circ = -6.283 = -2\pi$

$r = \cos 4\theta \Rightarrow 4\theta = \left\{ \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}, \frac{13\pi}{2}, \frac{15\pi}{2} \right\} \Rightarrow \theta = \left\{ \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8} \right\}$



$A = 16 \cdot \frac{1}{2} \int_0^{\pi/8} (\cos 4\theta)^2 d\theta$
 $\Rightarrow \cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$
 $\Rightarrow \cos^2(4\theta) = \frac{1}{2}(1 + \cos 8\theta)$

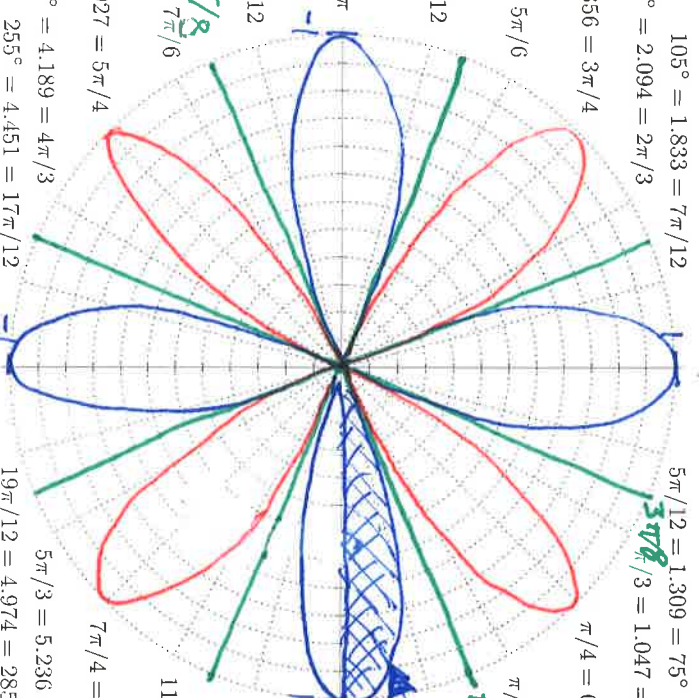
$= 8 \cdot \frac{1}{2} \int_0^{\pi/8} (1 + \cos 8\theta) d\theta$

$\int \cos ax dx = \frac{1}{a} \sin ax + C$

$= 4 \left[\theta + \frac{1}{8} \sin 8\theta \right]_0^{\pi/8}$

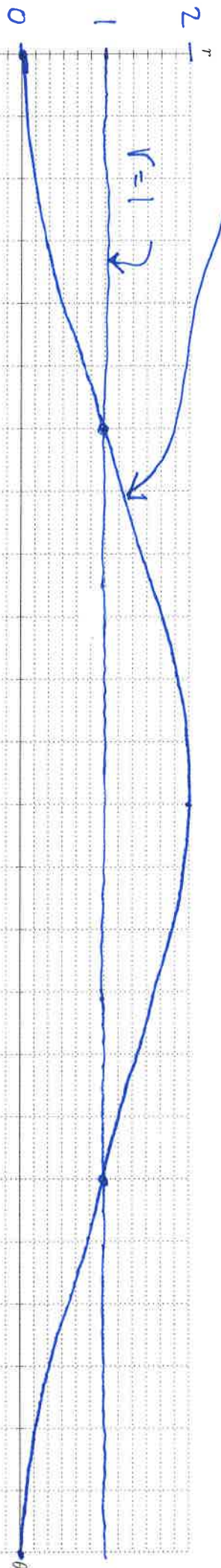
$= \frac{\pi}{2}$

- $105^\circ = 1.833 = 7\pi/12$
- $120^\circ = 2.094 = 2\pi/3$
- $150^\circ = 2.618 = 5\pi/6$
- $165^\circ = 2.880 = 11\pi/12$
- $180^\circ = 3.142 = \pi$
- $195^\circ = 3.403 = 13\pi/12$
- $210^\circ = 3.665 = 7\pi/6$
- $225^\circ = 3.927 = 5\pi/4$
- $240^\circ = 4.189 = 4\pi/3$
- $255^\circ = 4.451 = 17\pi/12$
- $270^\circ = 4.712 = 3\pi/2 = 270^\circ$
- $285^\circ = 4.974 = 285^\circ$
- $300^\circ = 5.236 = 300^\circ$
- $315^\circ = 5.498 = 315^\circ$
- $330^\circ = 5.760 = 330^\circ$
- $345^\circ = 6.021 = 345^\circ$
- $360^\circ = 6.283 = 360^\circ$



8.6.5

$$r = 1 - \cos \theta \Rightarrow \cos \theta = 1 \Rightarrow \theta = 0$$



0	0.000	0.262	0.524	0.785	1.047	1.309	1.571	1.833	2.094	2.356	2.618	2.880	3.142	3.403	3.665	3.927	4.189	4.451	4.712	4.974	5.236	5.498	5.760	6.021	6.283
0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°	0°

$$A = 2 \cdot \frac{1}{2} \int_{\pi/2}^{\pi} (1 - \cos \theta)^2 d\theta - \frac{\pi}{2} = \int_{\pi/2}^{\pi} (1 - 2\cos \theta + \cos^2 \theta) d\theta - \frac{\pi}{2}$$

$$= \int_{\pi/2}^{\pi} \left[\cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta) \right] d\theta - \frac{\pi}{2}$$

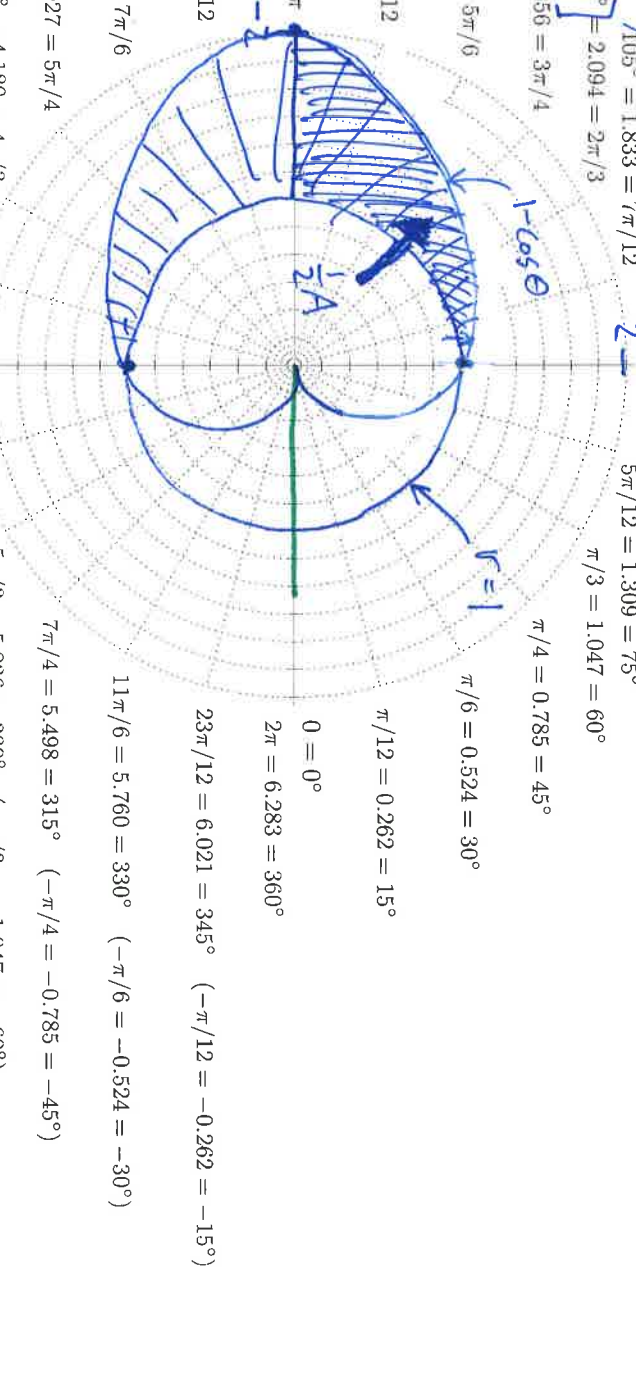
$$= \int_{\pi/2}^{\pi} \left(1 - 2\cos \theta + \frac{1}{2} + \frac{1}{2}\cos 2\theta \right) d\theta - \frac{\pi}{2} = \int_{\pi/2}^{\pi} \left(\frac{3}{2} - 2\cos \theta + \frac{1}{2}\cos 2\theta \right) d\theta - \frac{\pi}{2}$$

$$= \int_{\pi/2}^{\pi} \left[\frac{3}{2}\theta - 2\sin \theta + \frac{1}{4}\sin 2\theta \right]_{\pi/2}^{\pi} - \frac{\pi}{2}$$

$$= \left(\frac{3\pi}{2} \right) - \left(\frac{3 \cdot \frac{\pi}{2} - 2 + \frac{1}{4} \cdot 0 \right) - \frac{\pi}{2} = \frac{6}{4}\pi - \frac{3}{4}\pi - \frac{2\pi}{4} + 2 = \frac{\pi}{4} + 2$$

$$= \frac{6}{4}\pi - \frac{3}{4}\pi - \frac{2\pi}{4} + 2 = \frac{\pi}{4} + 2$$

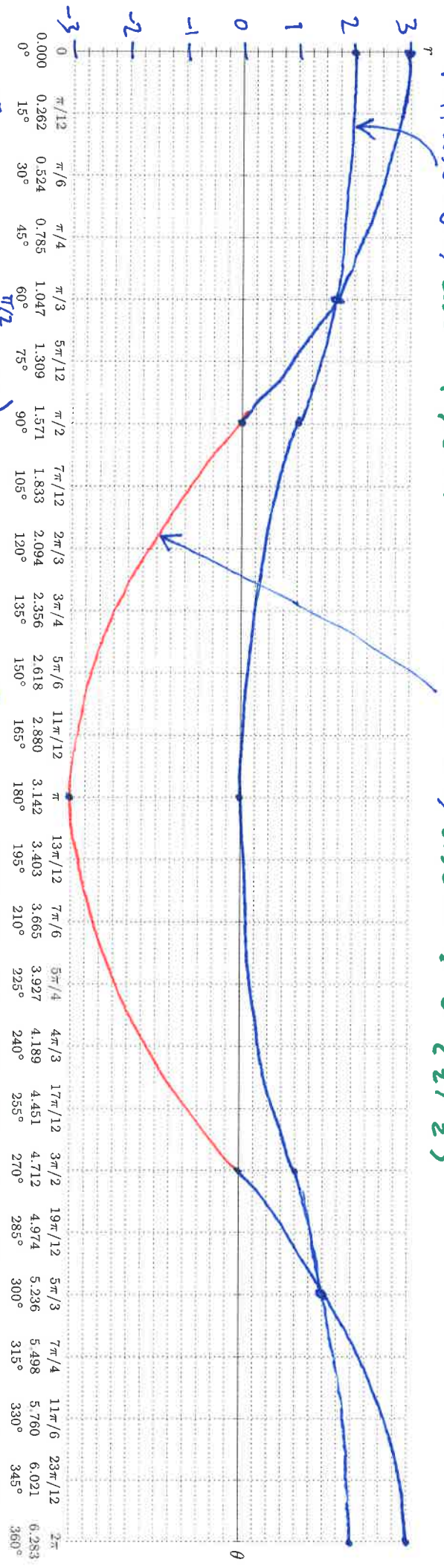
$$= \frac{\pi}{4} + 2$$



- $150^\circ = 2.618 = 5\pi/6$
- $165^\circ = 2.880 = 11\pi/12$
- $180^\circ = 3.142 = \pi$
- $195^\circ = 3.403 = 13\pi/12$
- $210^\circ = 3.665 = 7\pi/6$
- $225^\circ = 3.927 = 5\pi/4$
- $240^\circ = 4.189 = 4\pi/3$
- $255^\circ = 4.451 = 17\pi/12$
- $270^\circ = 4.712 = 3\pi/2$
- $285^\circ = 4.974 = 19\pi/12$
- $300^\circ = 5.236 = 5\pi/3$
- $315^\circ = 5.498 = 7\pi/4$
- $330^\circ = 5.760 = 11\pi/6$
- $345^\circ = 6.021 = 23\pi/12$
- $360^\circ = 6.283 = 2\pi$

$$r = 1 + \cos\theta = 0 \Rightarrow \cos\theta = -1 \Rightarrow \theta = \pi$$

$$r = 3\cos\theta = 0 \Rightarrow \cos\theta = 0 \Rightarrow \theta = \left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\}$$



$$A = 2 \cdot \frac{1}{2} \left(\int_{\pi/3}^{\pi} (1 + \cos\theta)^2 d\theta - 3^2 \int_{\pi/3}^{\pi/2} (\cos\theta)^2 d\theta \right)$$

$$= \int_{\pi/3}^{\pi} (1 + 2\cos\theta + \cos^2\theta) d\theta - 9 \int_{\pi/3}^{\pi/2} \cos^2\theta d\theta$$

$$= \int_{\pi/3}^{\pi} \left(1 + 2\cos\theta + \frac{1}{2} + \frac{1}{2}\cos(2\theta) \right) d\theta - 9 \int_{\pi/3}^{\pi/2} \left(\frac{1}{2} + \frac{1}{2}\cos(2\theta) \right) d\theta$$

$$= \left[\frac{3}{2}\theta + 2\sin\theta + \frac{1}{4}\sin(2\theta) \right]_{\pi/3}^{\pi} - \frac{9}{2} \left[\theta + \frac{1}{2}\sin(2\theta) \right]_{\pi/3}^{\pi/2}$$

$$= \frac{3}{2} \cdot \pi + 2\sin\pi + \frac{1}{4}\sin(2\pi) - \frac{3}{2} \cdot \frac{\pi}{3} - 2\sin\frac{\pi}{3} - \frac{1}{4}\sin\frac{2\pi}{3} - \frac{9}{2} \left(\frac{\pi}{2} + \frac{1}{2}\sin\pi - \frac{\pi}{3} - \frac{1}{2}\sin\frac{2\pi}{3} \right)$$

$$= \frac{\pi}{4}$$

$$\left[\cos^2\theta = \frac{1}{2}(1 + \cos(2\theta)) \right]$$

$$\left[\cos ax \cos bx = \frac{1}{2}(\cos(ax+bx) + \cos(ax-bx)) \right]$$

- 120° = 2.094 = 2π/3
- 105° = 1.833 = 7π/12
- 5π/12 = 1.309 = 75°
- π/3 = 1.047 = 60°
- π/4 = 0.785 = 45°
- π/6 = 0.524 = 30°
- π/12 = 0.262 = 15°
- 0 = 0°
- 2π = 6.283 = 360°
- 23π/12 = 6.021 = 345°
- (-π/12 = -0.262 = -15°)
- 11π/6 = 5.760 = 330°
- (-π/6 = -0.524 = -30°)
- 7π/4 = 5.498 = 315°
- (-π/4 = -0.785 = -45°)
- 5π/3 = 5.236 = 300°
- (-π/3 = -1.047 = -60°)
- 19π/12 = 4.974 = 285°
- (-5π/12 = -1.309 = -75°)
- 4.712 = 3π/2 = 270°
- 19π/12 = 4.974 = 285°
- (-5π/12 = -1.309 = -75°)
- 4.712 = 3π/2 = 270°
- (-1.571 = -π/2 = -90°)

$$1 + \cos\theta = 3\cos\theta$$

$$-2\cos\theta = -1$$

$$\cos\theta = \frac{1}{2}$$

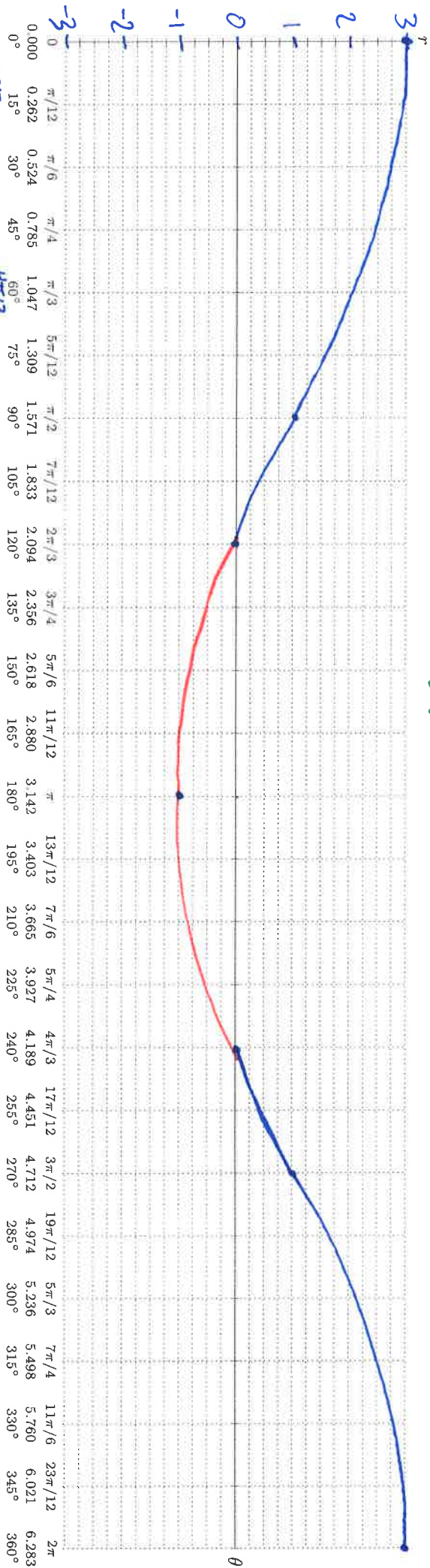
$$\theta = \left\{ \frac{\pi}{3}, \frac{5\pi}{3} \right\}$$

- 210° = 3.665 = 7π/6
- 225° = 3.927 = 5π/4
- (-120° = -2.094 = -2π/3)
- 240° = 4.189 = 4π/3
- (-105° = -1.833 = -7π/12)
- 255° = 4.451 = 17π/12
- 4.712 = 3π/2 = 270°
- (-1.571 = -π/2 = -90°)
- 5π/3 = 5.236 = 300°
- (-π/3 = -1.047 = -60°)
- 19π/12 = 4.974 = 285°
- (-5π/12 = -1.309 = -75°)
- 4.712 = 3π/2 = 270°
- (-1.571 = -π/2 = -90°)

- 11π/6 = 5.760 = 330°
- (-π/6 = -0.524 = -30°)
- 7π/4 = 5.498 = 315°
- (-π/4 = -0.785 = -45°)
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- (-π/3 = -1.047 = -60°)
- 19π/12 = 4.974 = 285°
- (-5π/12 = -1.309 = -75°)
- 4.712 = 3π/2 = 270°
- (-1.571 = -π/2 = -90°)

8.6.9

$r = 1 + 2\cos\theta = 0 \Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = \{ \frac{2\pi}{3}, \frac{4\pi}{3} \}$



$A = \frac{1}{2} \int_{2\pi/3}^{4\pi/3} (1 + 2\cos\theta)^2 d\theta = \frac{1}{2} \int_{2\pi/3}^{4\pi/3} (1 + 4\cos\theta + 4\cos^2\theta) d\theta$

$= \frac{1}{2} \int_{2\pi/3}^{4\pi/3} (1 + 4\cos\theta + 2 + 2\cos(2\theta)) d\theta = \frac{1}{2} \int_{2\pi/3}^{4\pi/3} (3 + 4\cos\theta + 2\cos(2\theta)) d\theta$

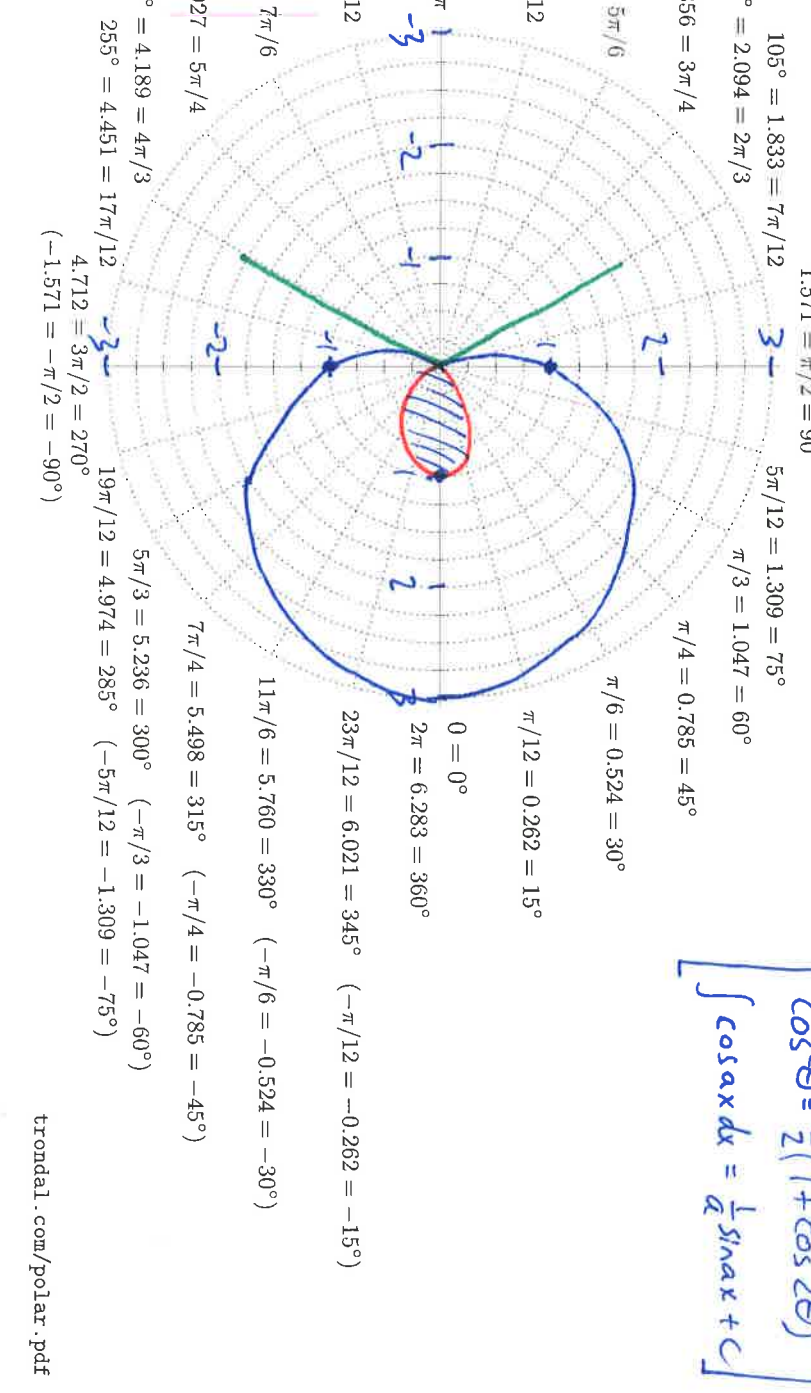
$= \frac{1}{2} [3\theta + 4\sin\theta + \sin(2\theta)]_{2\pi/3}^{4\pi/3}$

$= \frac{1}{2} (3 \cdot \frac{4\pi}{3} + 4\sin\frac{4\pi}{3} + \sin\frac{8\pi}{3}) - \frac{1}{2} (3 \cdot \frac{2\pi}{3} + 4\sin\frac{2\pi}{3} + \sin\frac{4\pi}{3})$

$= \frac{1}{2} (2\pi - 3\sqrt{3}) = \pi - \frac{3\sqrt{3}}{2}$

- $165^\circ = 2.880 = 11\pi/12$
- $180^\circ = -3.142 = -\pi$
- $195^\circ = 3.403 = 13\pi/12$
- $210^\circ = -2.618 = -5\pi/6$
- $225^\circ = 3.665 = 7\pi/6$
- $240^\circ = -2.356 = -3\pi/4$
- $255^\circ = 3.927 = 5\pi/4$
- $270^\circ = -1.571 = -\pi/2 = 90^\circ$
- $285^\circ = 4.189 = 4\pi/3$
- $300^\circ = -1.047 = -2\pi/3$
- $315^\circ = 4.451 = 17\pi/12$
- $330^\circ = -0.785 = -\pi/4$
- $345^\circ = 4.712 = 3\pi/2 = 270^\circ$
- $360^\circ = -1.571 = -\pi/2 = -90^\circ$

$\int \cos^2\theta = \frac{1}{2}(1 + \cos(2\theta))$
 $\int \cos\theta dx = \frac{1}{a} \sin ax + C$



8.6.11

8.6.12

$$r = \theta^2, \quad 0 \leq \theta \leq \pi$$

$$(r')^2 = (2\theta)^2 = 4\theta^2$$

$$S = \int_0^{\pi} \sqrt{(r')^2 + r^2} d\theta$$

$$= \int_0^{\pi} \sqrt{4\theta^2 + \theta^4} d\theta$$

$$= \int_0^{\pi} \theta \sqrt{4 + \theta^2} d\theta$$

$$\left[\begin{array}{l} u = 4 + \theta^2 \\ du = 2\theta d\theta \\ d\theta = \frac{du}{2\theta} \end{array} \right]$$

$$= \int_{u(0)}^{u(\pi)} \theta \sqrt{u} \cdot \frac{du}{2\theta}$$

$$= \frac{1}{2} \int_4^{4+\pi^2} u^{1/2} du$$

$$= \frac{1}{3} \left[u^{3/2} \right]_4^{4+\pi^2}$$

$$= \frac{1}{3} \left((4+\pi^2)^{3/2} - 8 \right)$$