8.5.25

$r = f(\theta) = \sqrt{3}\cos\theta$

$r = g(\theta) = \sin\theta$

1) $f(\theta) = 0$

$\sqrt{3}\cos\theta = 0$

$\cos\theta = 0$

$\theta = \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$ ok.

$g(\theta) = 0$

$\sin\theta = 0$

$\theta = \{0, \pi\}$ ok.

$\Rightarrow$ Skjæring i origo; $[0, 0]$

2) $f(\theta) = g(\theta)$

$\sqrt{3}\cos\theta = \sin\theta$

$\tan\theta = \sqrt{3}$

$\theta = \left\{\frac{\pi}{3}, \frac{4\pi}{3}\right\}$

$g\left(\frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$g\left(\frac{4\pi}{3}\right) = \sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

$\Rightarrow$ Skjæring i $\left[\frac{\sqrt{3}}{2}, \frac{\pi}{3}\right]$

og $\left[-\frac{\sqrt{3}}{2}, \frac{4\pi}{3}\right]$

$= \left[\frac{\sqrt{3}}{3}, \frac{4\pi}{3} - \pi\right]$

$= \left[\frac{\sqrt{3}}{3}, \frac{\pi}{3}\right]$

dvs, samme punkt

3) $f(\theta + (2k+1)\pi) = -g(\theta)$

$\sqrt{3}\cos(\theta + (2k+1)\pi) = -\sin\theta$

$\sqrt{3}\cos(\theta + 2k\pi + \pi) = -\sin\theta$

$\sqrt{3}\cos(\theta + \pi) = -\sin\theta$

$\sqrt{3}(-\cos(\theta)) = -\sin\theta$

$\sqrt{3} = \tan\theta$

$\theta = \left\{\frac{\pi}{3}, \frac{4\pi}{3}\right\}$

dvs samme løsninger som fra 2)

8.5.27

$r = f(\theta) = 1 + \cos(\theta)$

$r = g(\theta) = 3\cos(\theta)$

1) $f(\theta) = 0$

$1 + \cos(\theta) = 0$

$\cos(\theta) = -1$

$\theta = \pi$ ok.

$g(\theta) = 0$

$3\cos(\theta) = 0$

$\cos(\theta) = 0$

$\theta = \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$ ok.

$\Rightarrow$ Skjæring i origo; $[0,0]$

2) $f(\theta) = g(\theta)$

$1 + \cos(\theta) = 3\cos(\theta)$

$-2\cos(\theta) = -1$

$\cos(\theta) = \frac{1}{2}$

$\theta = \left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

$g\left(\frac{\pi}{3}\right) = 3 \cdot \cos\left(\frac{\pi}{3}\right) = \frac{3}{2}$

$g\left(\frac{5\pi}{3}\right) = 3 \cdot \cos\left(\frac{5\pi}{3}\right) = \frac{3}{2}$

$\Rightarrow$ Skjæring i $\left[\frac{3}{2}, \frac{\pi}{3}\right]$

og $\left[\frac{3}{2}, \frac{5\pi}{3}\right]$

3) $f(\theta + (2k+1)\pi) = -g(\theta)$

$1 + \cos(\theta + (2k+1)\pi) = -3\cos(\theta)$

$1 + \cos(\theta + 2k\pi + \pi) = -3\cos(\theta)$

$1 + \cos(\theta + \pi) = -3\cos(\theta)$

$1 - \cos(\theta) = -3\cos(\theta)$

$2\cos(\theta) = -1$

$\cos(\theta) = -\frac{1}{2}$

$\theta = \left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

$g\left(\frac{2\pi}{3}\right) = -\frac{3}{2}$

$g\left(\frac{4\pi}{3}\right) = -\frac{3}{2}$

$\Rightarrow$ Skjæring i

$\left[-\frac{3}{2}, \frac{2\pi}{3}\right] = \left[\frac{3}{2}, \frac{2\pi}{3} + \pi\right] = \left[\frac{3}{2}, \frac{5\pi}{3}\right]$

og

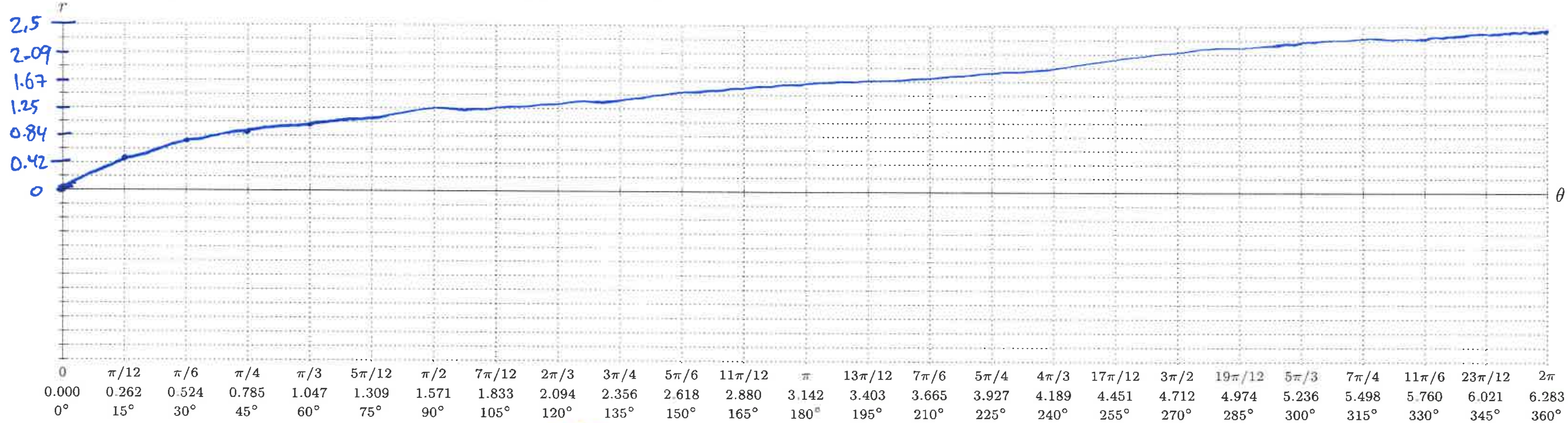
$\left[-\frac{3}{2}, \frac{4\pi}{3}\right] = \left[\frac{3}{2}, \frac{4\pi}{3} - \pi\right] = \left[\frac{3}{2}, \frac{\pi}{3}\right]$

dvs. samme løsninger som fra 2)

8.6.1

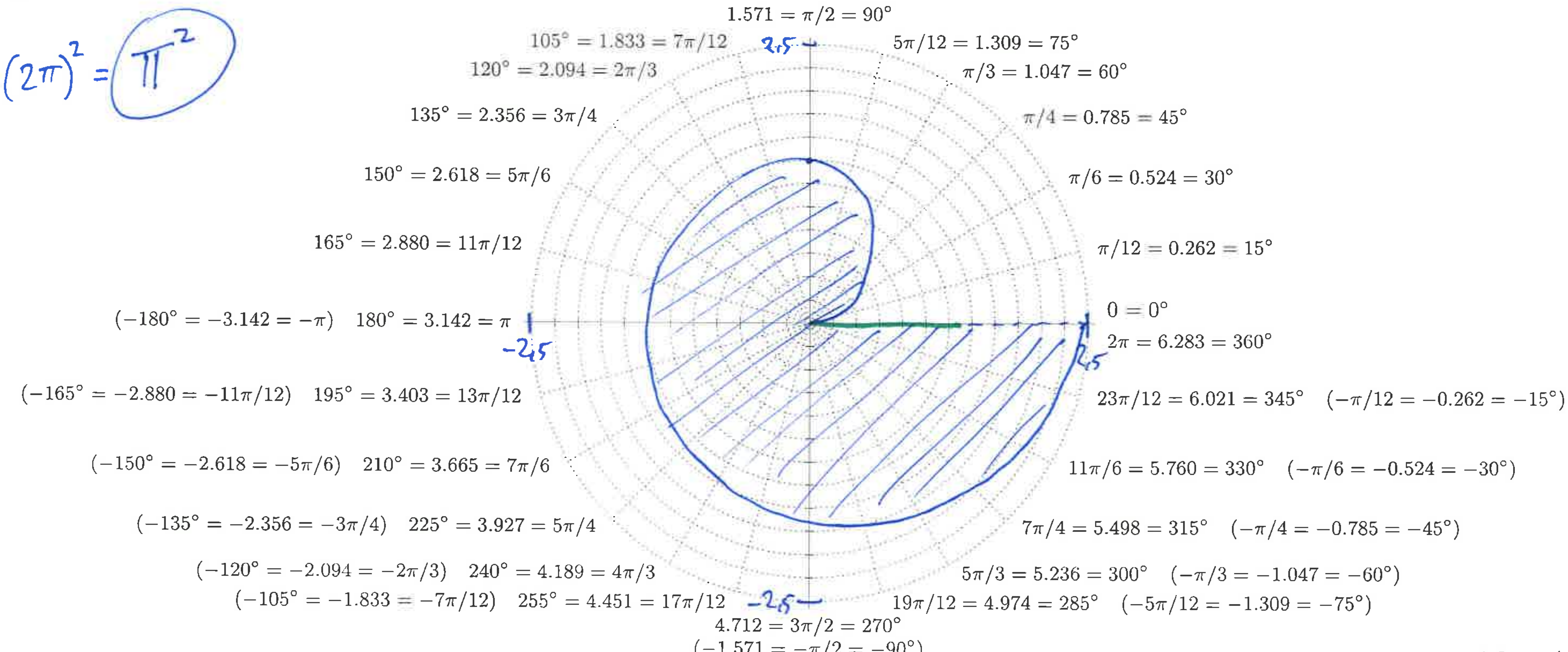
$r = \sqrt{\theta}$

$r = \sqrt{\theta} = 0 \Rightarrow \theta = 0$



$$A = \frac{1}{2}\int_0^{2\pi} (\sqrt{\theta})^2 \, d\theta = \frac{1}{2}\int_0^{2\pi} \theta \, d\theta = \frac{1}{2}\left[\frac{1}{2}\theta^2\right]_0^{2\pi}$$

$$= \frac{1}{4}\cdot(2\pi)^2 = \pi^2$$



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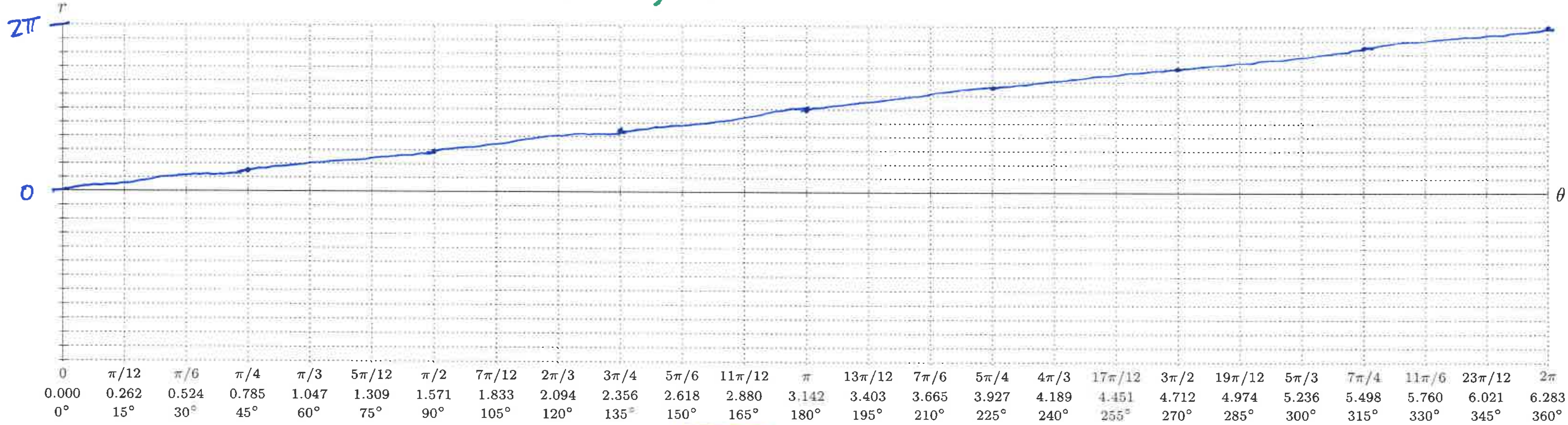
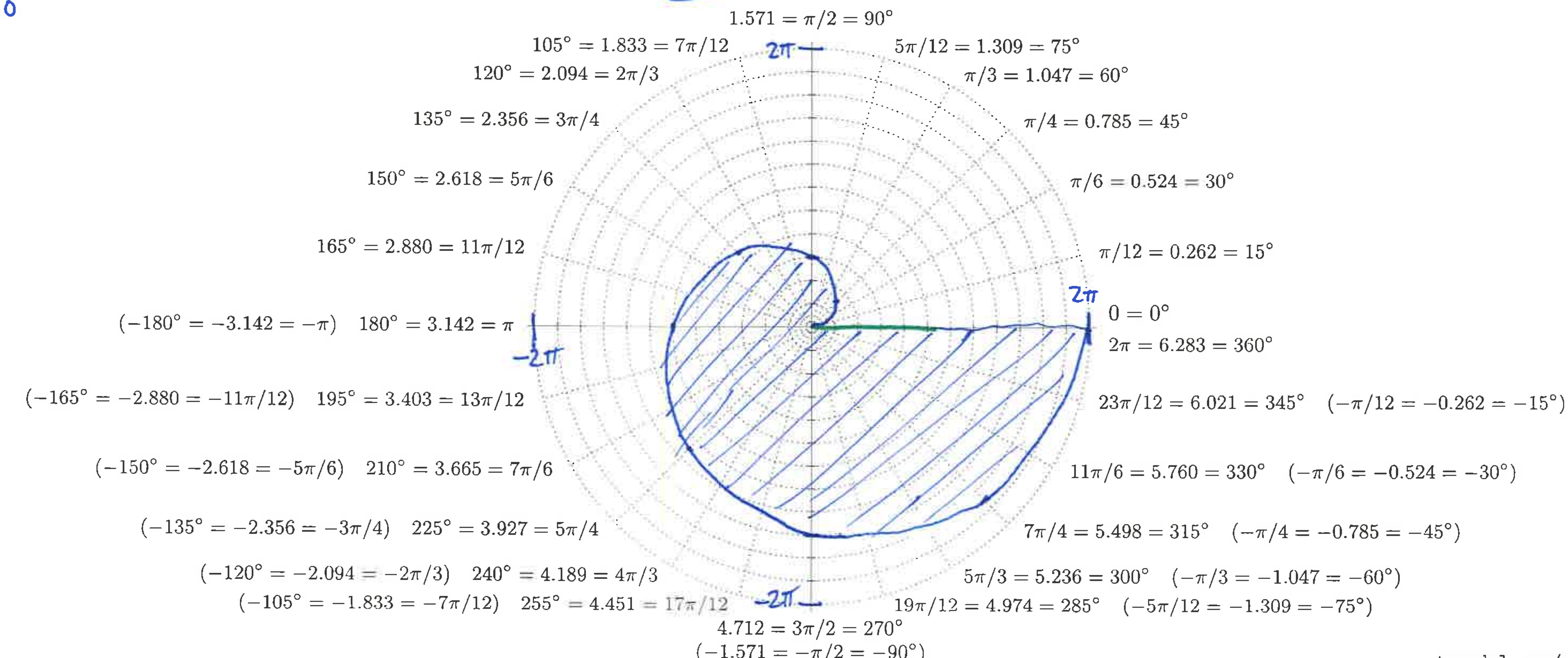
8.6.2

$r = \theta$

$r = \theta = 0 \Rightarrow \theta = 0$



$$A = \frac{1}{2}\int_0^{2\pi} \theta^2 \, d\theta = \frac{1}{2}\left[\frac{1}{3}\theta^3\right]_0^{2\pi} = \frac{1}{6}\cdot(2\pi)^3 = \frac{8}{6}\pi^3 = \frac{4}{3}\pi^3$$



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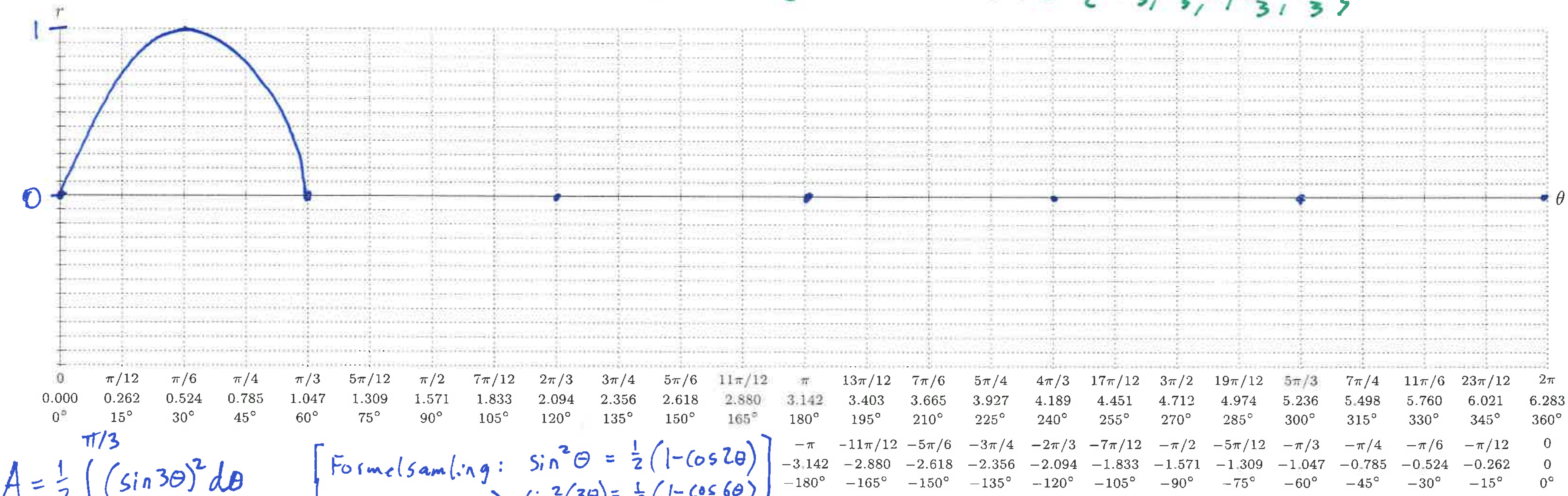
$r = \sin 3\theta$

$r = \sin 3\theta = 0 \Rightarrow 3\theta = \{0, \pi, 2\pi, 3\pi, 4\pi, 5\pi\} \Rightarrow \theta = \{0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}\}$



$$A = \frac{1}{2}\int_0^{\pi/3} (\sin 3\theta)^2 \, d\theta$$

[Formelsamling: $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

$\Rightarrow \sin^2(3\theta) = \frac{1}{2}(1-\cos 6\theta)$]

$$= \frac{1}{2}\int_0^{\pi/3} \frac{1}{2}(1-\cos 6\theta)\, d\theta$$

[Formelsamling:

$\int \cos ax \, dx = \frac{1}{a}\sin ax + C$]

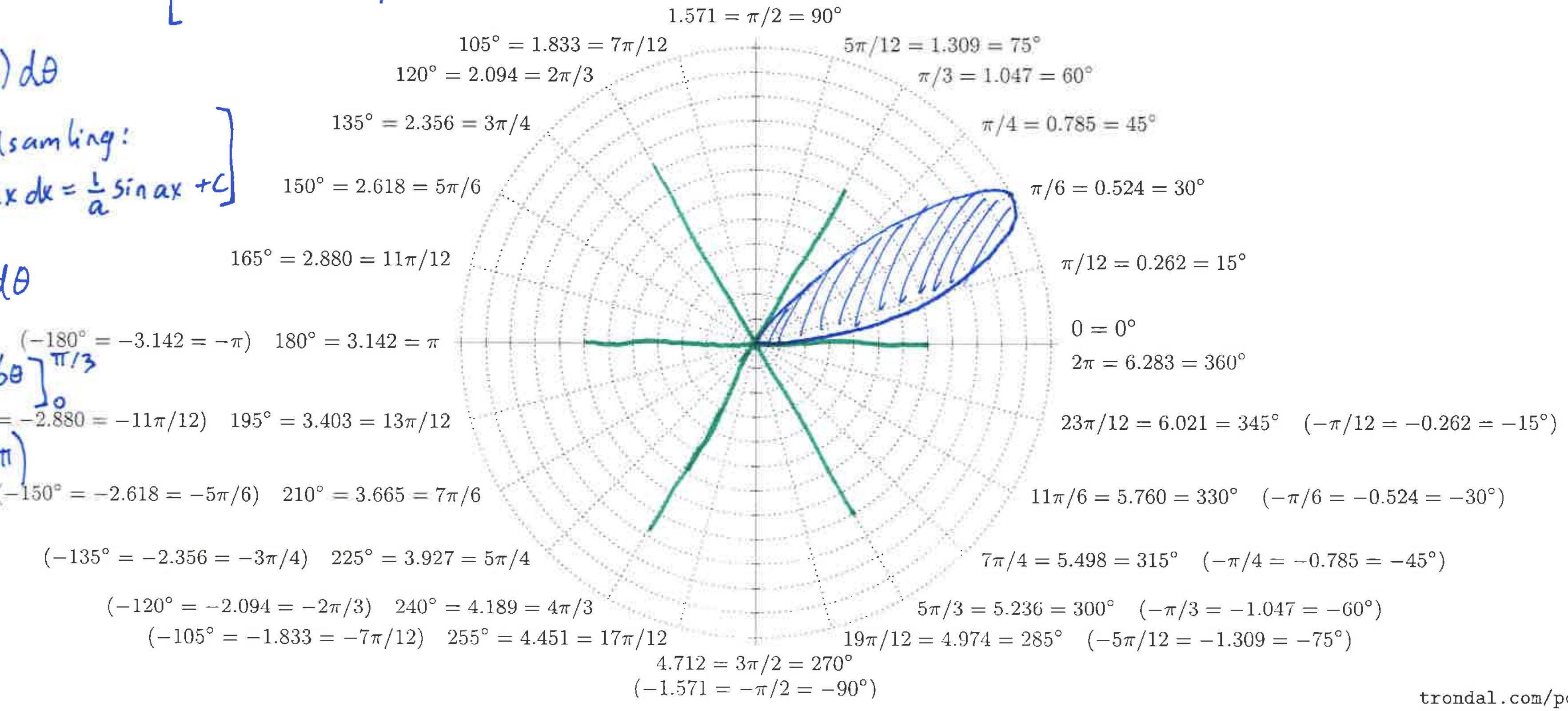
$$= \frac{1}{4}\int_0^{\pi/3} (1-\cos 6\theta)\, d\theta$$

$$= \frac{1}{4}\left[\theta - \frac{1}{6}\sin 6\theta\right]_0^{\pi/3}$$

$$= \frac{1}{4}\left(\frac{\pi}{3} - \frac{1}{6}\cdot\sin 2\pi\right)$$

$$= \frac{\pi}{12}$$

8.6.4



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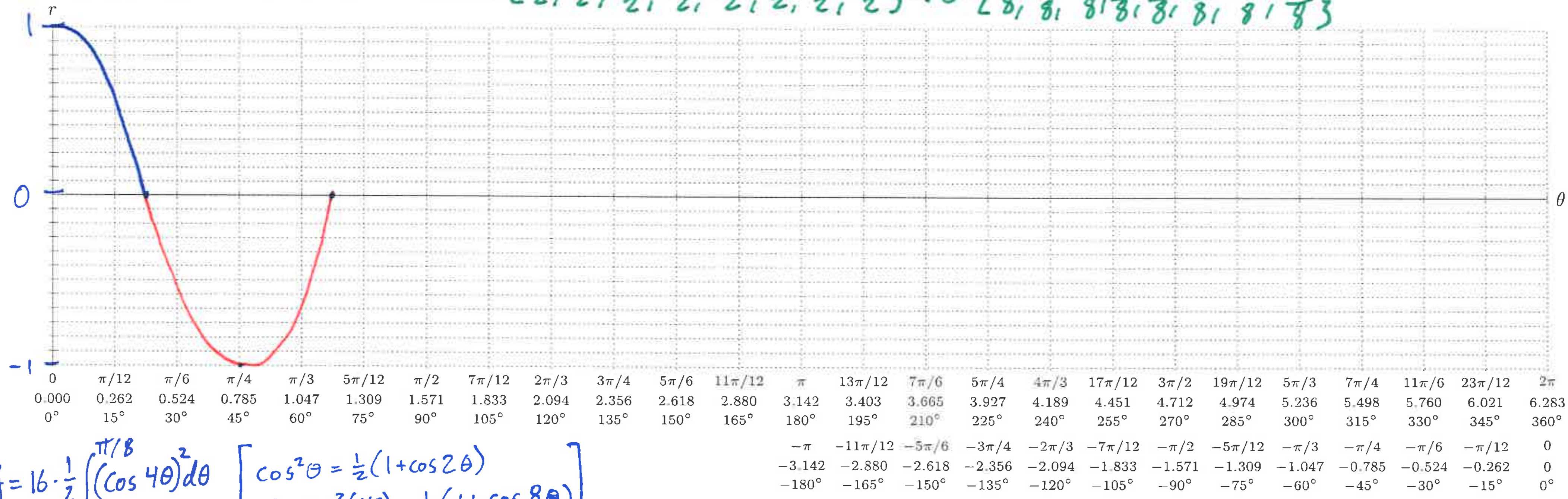
8.6.5

$r = \cos 4\theta$

$r = \cos 4\theta = 0 \Rightarrow 4\theta = \left\{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}, \frac{13\pi}{2}, \frac{15\pi}{2}\right\} \Rightarrow \theta = \left\{\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}\right\}$



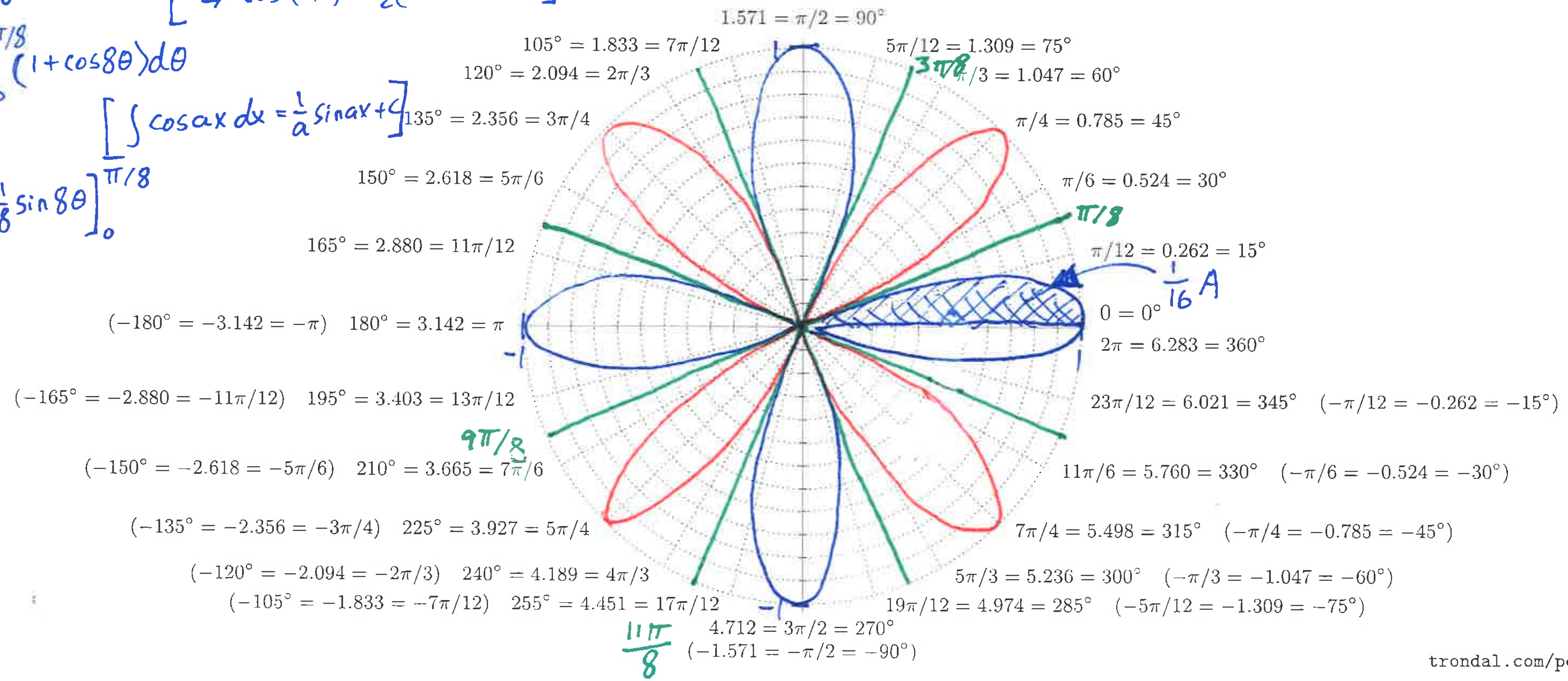
$$A = 16 \cdot \frac{1}{2}\int_0^{\pi/8} (\cos 4\theta)^2 \, d\theta \quad \left[\cos^2\theta = \frac{1}{2}(1+\cos 2\theta) \Rightarrow \cos^2(4\theta) = \frac{1}{2}(1+\cos 8\theta)\right]$$

$$= 8 \cdot \frac{1}{2} \cdot \int_0^{\pi/8} (1+\cos 8\theta)\, d\theta$$

$$\left[\int \cos ax \, dx = \frac{1}{a}\sin ax + C\right]$$

$$= 4\left[\theta + \frac{1}{8}\sin 8\theta\right]_0^{\pi/8}$$

$$= \frac{\pi}{2}$$



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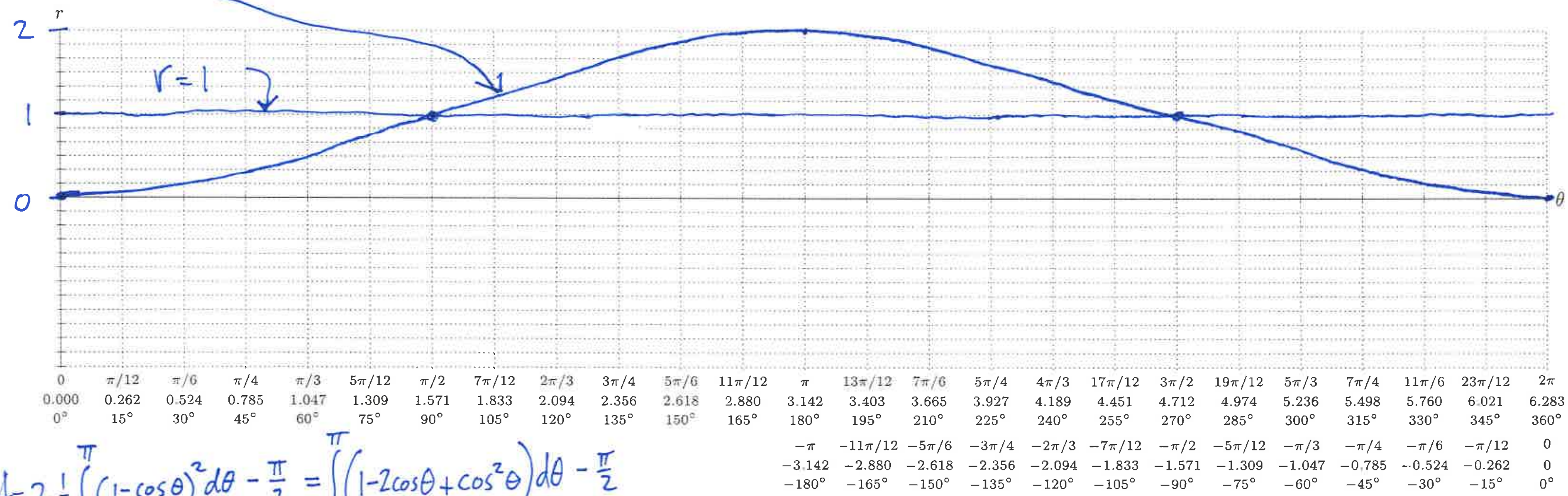
$r = 1-\cos\theta = 0 \Rightarrow \cos\theta = 1 \Rightarrow \theta = 0$



$$A = 2\cdot\frac{1}{2}\int_{\pi/2}^{\pi}(1-\cos\theta)^2\,d\theta - \frac{\pi}{2} = \int_{\pi/2}^{\pi}\left(1-2\cos\theta+\cos^2\theta\right)d\theta - \frac{\pi}{2}$$

$$\left[\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)\right]$$

$$= \int_{\pi/2}^{\pi}\left(1-2\cos\theta+\frac{1}{2}+\frac{1}{2}\cos 2\theta\right)d\theta - \frac{\pi}{2} \quad \left[\int \cos ax\,dx = \frac{1}{a}\sin ax + C\right]$$

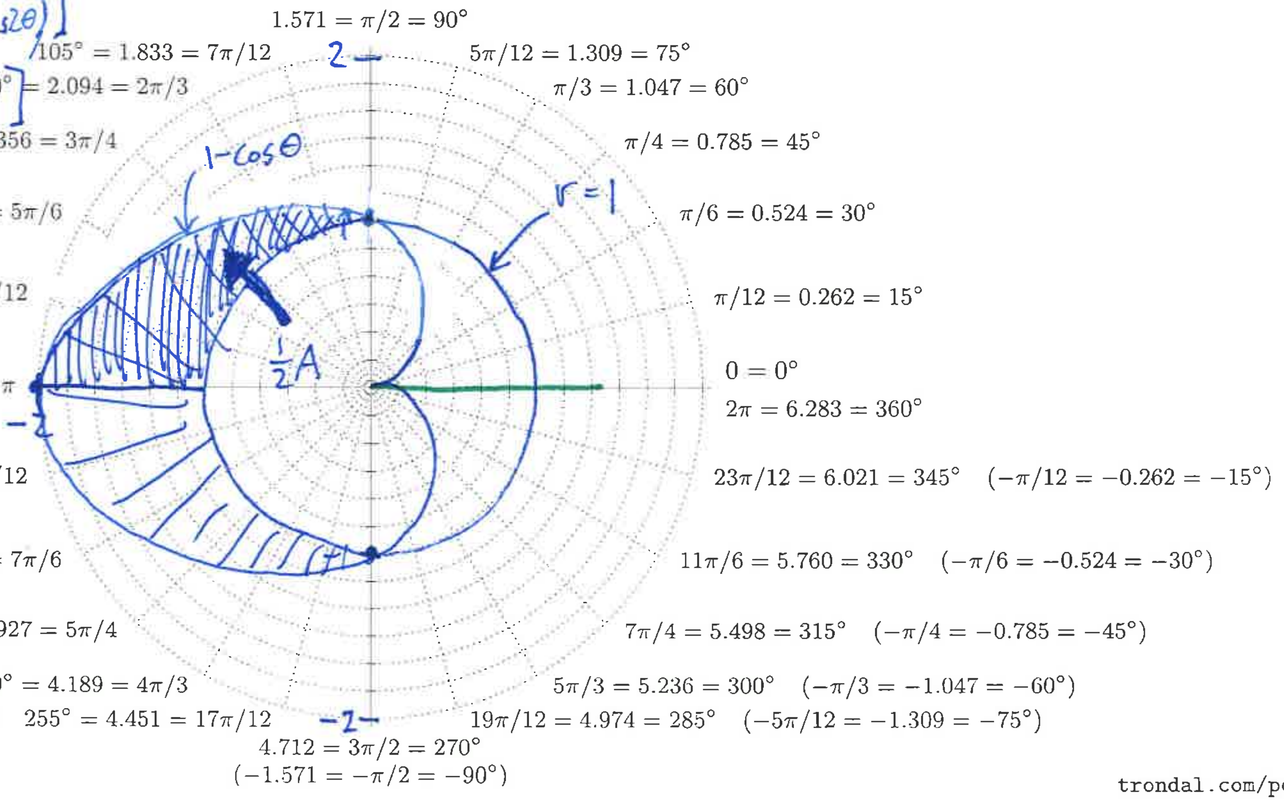
$$= \int_{\pi/2}^{\pi}\left(\frac{3}{2}-2\cos\theta+\frac{1}{2}\cos 2\theta\right)d\theta - \frac{\pi}{2}$$

$$= \left[\frac{3}{2}\theta - 2\sin\theta + \frac{1}{2}\cdot\frac{1}{2}\sin 2\theta\right]_{\pi/2}^{\pi} - \frac{\pi}{2}$$

$$= \left(\frac{3\pi}{2}\right) - \left(\frac{3}{2}\cdot\frac{\pi}{2} - 2 + \frac{1}{4}\cdot 0\right) - \frac{\pi}{2}$$

$$= \frac{6}{4}\pi - \frac{3}{4}\pi - \frac{2\pi}{4} + 2$$

8.6.7 $= \frac{\pi}{4} + 2$



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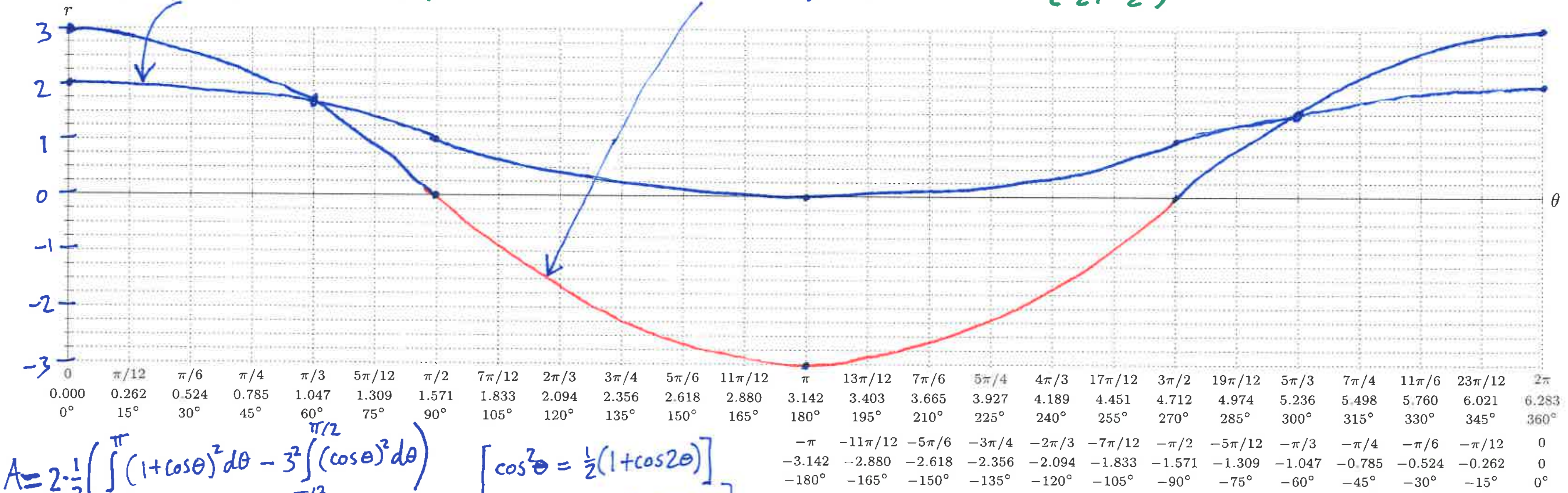
8.6.9

$r = 1+\cos\theta = 0 \Rightarrow \cos\theta = -1 \Rightarrow \theta = \pi$

$r = 3\cos\theta = 0 \Rightarrow \cos\theta = 0 \Rightarrow \theta = \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$



$$A = 2\cdot\frac{1}{2}\left(\int_{\pi/3}^{\pi}(1+\cos\theta)^2\,d\theta - 3^2\int_{\pi/3}^{\pi/2}(\cos\theta)^2\,d\theta\right)$$

$$= \int_{\pi/3}^{\pi}\left(1+2\cos\theta+\cos^2\theta\right)d\theta - 9\int_{\pi/3}^{\pi/2}\cos^2\theta\,d\theta$$

$$= \int_{\pi/3}^{\pi}\left(1+2\cos\theta+\frac{1}{2}+\frac{1}{2}\cos 2\theta\right)d\theta - 9\int_{\pi/3}^{\pi/2}\left(\frac{1}{2}+\frac{1}{2}\cos 2\theta\right)d\theta$$

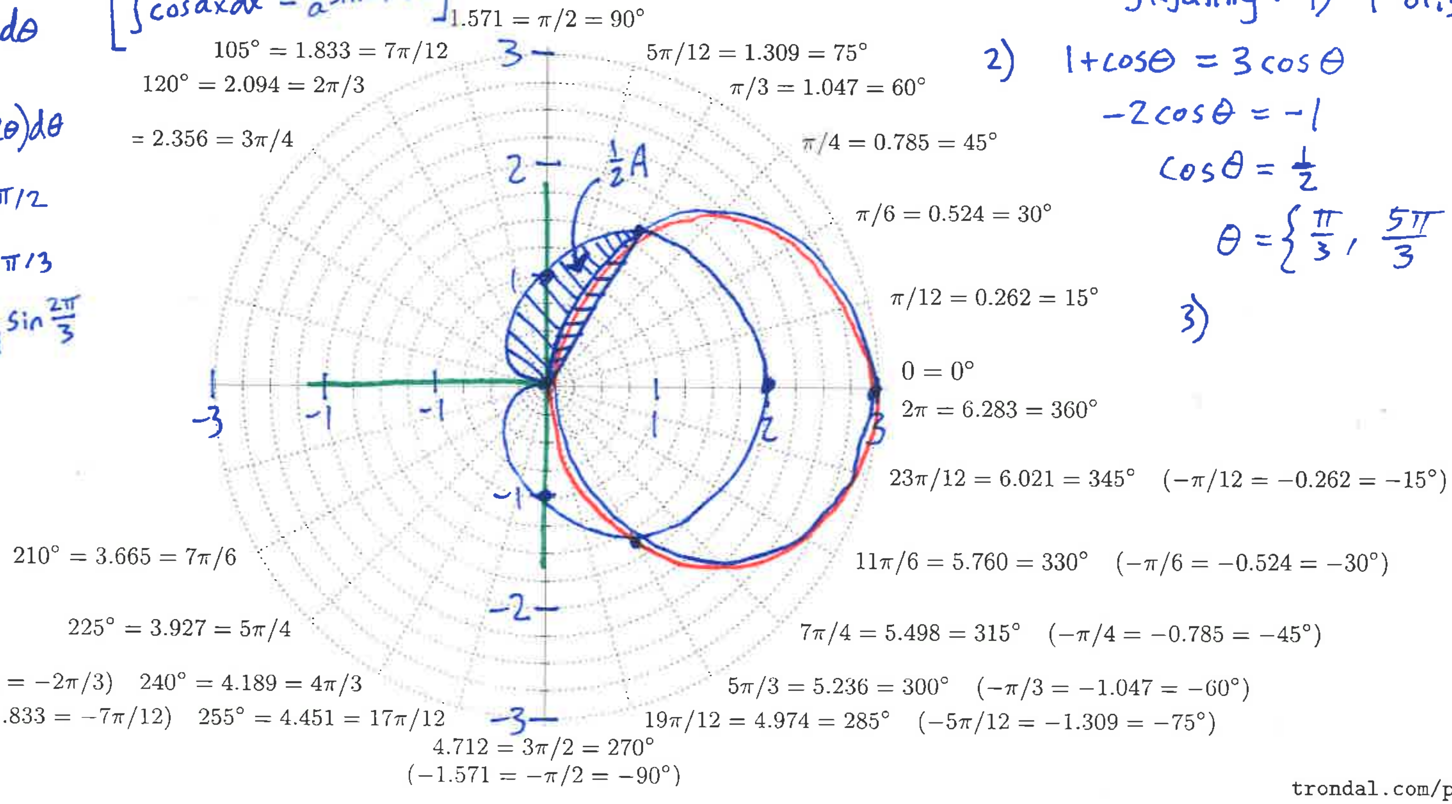
$$= \left[\frac{3}{2}\theta + 2\sin\theta + \frac{1}{4}\sin 2\theta\right]_{\pi/3}^{\pi} - \frac{9}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right]_{\pi/3}^{\pi/2}$$

$$= \frac{3}{2}\cdot\pi + \cancel{2\sin\pi} + \cancel{\frac{1}{4}\sin 2\pi} - \frac{3}{2}\cdot\frac{\pi}{3} - 2\sin\frac{\pi}{3} - \frac{1}{4}\sin\frac{2\pi}{3}$$
$$- \frac{9}{2}\left(\frac{\pi}{2} + \cancel{\frac{1}{2}\sin\pi} - \frac{\pi}{3} - \frac{1}{2}\sin\frac{2\pi}{3}\right)$$

$$= \frac{\pi}{4}$$

$\left[\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)\right]$

$\left[\int \cos ax\,dx = \frac{1}{a}\sin ax + C\right]$



Skjæring: 1) I origo

2) $1+\cos\theta = 3\cos\theta$

$-2\cos\theta = -1$

$\cos\theta = \frac{1}{2}$

$\theta = \left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

3)

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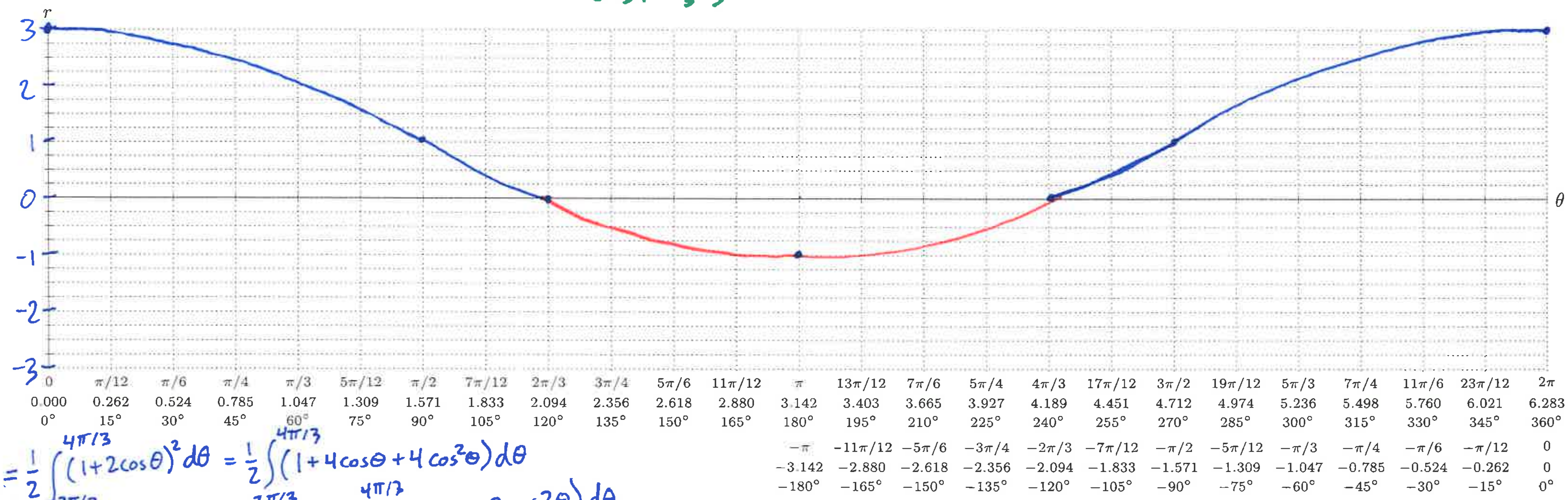
$r = 1 + 2\cos\theta = 0 \Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = \left\{ \frac{2\pi}{3}, \frac{4\pi}{3} \right\}$



$$A = \frac{1}{2}\int_{2\pi/3}^{4\pi/3} (1+2\cos\theta)^2 \, d\theta = \frac{1}{2}\int_{2\pi/3}^{4\pi/3} (1 + 4\cos\theta + 4\cos^2\theta)\, d\theta$$

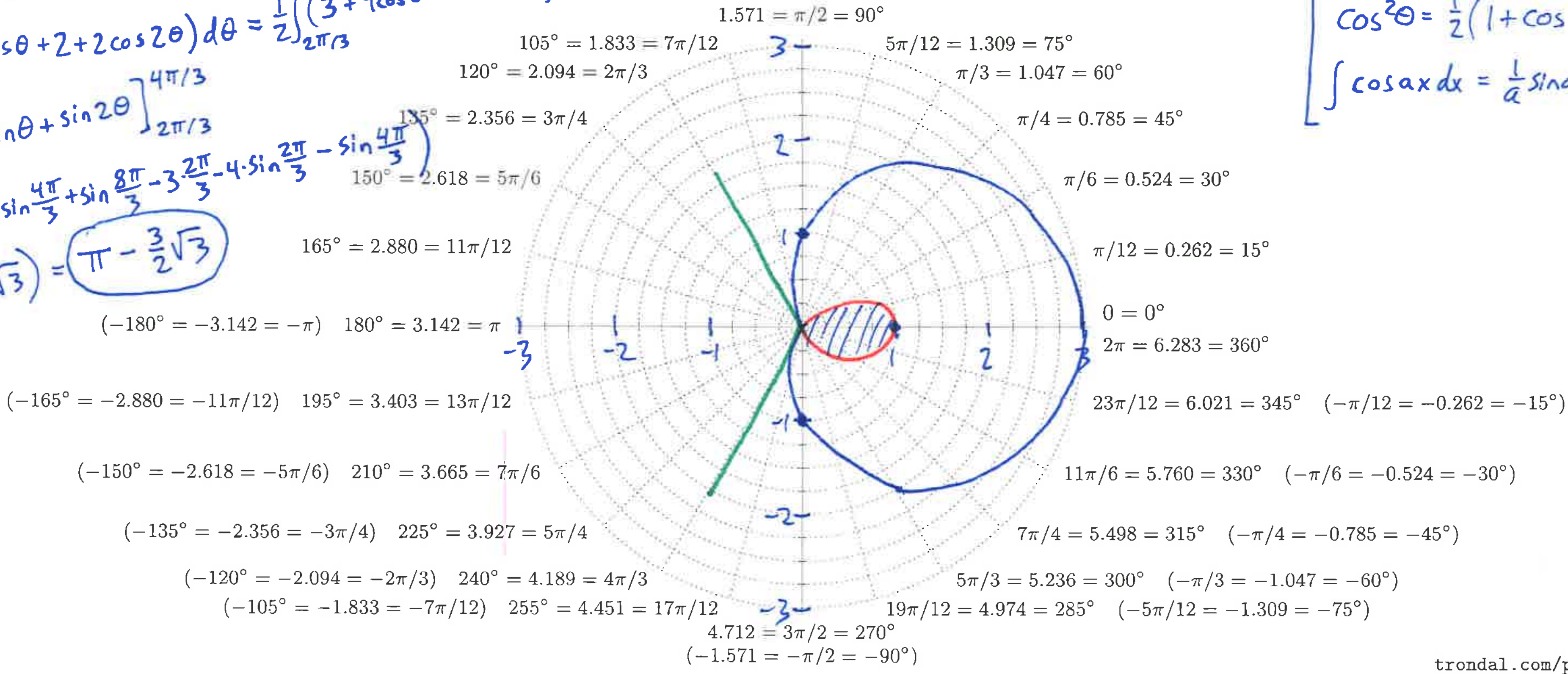
$$= \frac{1}{2}\int_{2\pi/3}^{4\pi/3} (1 + 4\cos\theta + 2 + 2\cos 2\theta)\, d\theta = \frac{1}{2}\int_{2\pi/3}^{4\pi/3} (3 + 4\cos\theta + 2\cos 2\theta)\, d\theta$$

$$= \frac{1}{2}\Big[3\theta + 4\sin\theta + \sin 2\theta\Big]_{2\pi/3}^{4\pi/3}$$

$$= \frac{1}{2}\left(3\cdot\frac{4\pi}{3} + 4\sin\frac{4\pi}{3} + \sin\frac{8\pi}{3} - 3\cdot\frac{2\pi}{3} - 4\cdot\sin\frac{2\pi}{3} - \sin\frac{4\pi}{3}\right)$$

$$= \frac{1}{2}\left(2\pi - 3\sqrt{3}\right) = \pi - \frac{3}{2}\sqrt{3}$$

$$\left[\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta) \qquad \int \cos ax \, dx = \frac{1}{a}\sin ax + C \right]$$



8.6.11

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8.6.12

$r = \theta^2, \quad 0 \le \theta \le \pi$

$(r')^2 = (2\theta)^2 = 4\theta^2$

$$S = \int_0^{\pi} \sqrt{(r')^2 + r^2}\, d\theta$$

$$= \int_0^{\pi} \sqrt{4\theta^2 + \theta^4}\, d\theta$$

$$= \int_0^{\pi} \theta\sqrt{4+\theta^2}\, d\theta$$

$$\left[\begin{array}{l} u = 4+\theta^2 \\ du = 2\theta\, d\theta \\ d\theta = \dfrac{du}{2\theta} \end{array}\right]$$

$$= \int_{u(0)}^{u(\pi)} \cancel{\theta}\sqrt{u} \cdot \frac{du}{2\cancel{\theta}}$$

$$= \frac{1}{2}\int_4^{4+\pi^2} u^{1/2}\, du$$

$$= \frac{1}{3}\Big[u^{3/2}\Big]_4^{4+\pi^2}$$

$$= \frac{1}{3}\left((4+\pi^2)^{3/2} - 8\right)$$