10.1.1

$$\sqrt{2^2+1^2+2^2} = \sqrt{9} = 3$$

10.1.2

$$\sqrt{(-1-1)^2+(-1-1)^2+(-1-1)^2} = \sqrt{12} = 2\sqrt{3}$$

10.1.3

$$\sqrt{(0-1)^2+(2-1)^2+(-2-0)^2} = \sqrt{6}$$

10.1.4

$$\sqrt{(-2-3)^2+(3-8)^2+(-6+1)^2}$$

$$= \sqrt{5^2+5^2+5^2} = \sqrt{75} = 5\sqrt{3}$$

10.1.5

a) På xy-planet er $z=0$, så avstanden det er snakk om er mellom (x,y,z) og $(x,y,0)$ og er lik $\sqrt{(x-x)^2+(y-y)^2+(z-0)^2}$

$$= \sqrt{z^2} = |z|$$

b) Avstand mellom (x,y,z) og $(x,0,0)$ er lik

$$\sqrt{(x-x)^2+(y-0)^2+(z-0)^2}$$

$$= \sqrt{y^2+z^2}$$

10.1.6

$A = (1,2,3)$, $B = (4,0,5)$, $C = (3,6,4)$

$$|AB| = \sqrt{(4-1)^2+(0-2)^2+(5-3)^2} = \sqrt{17}$$

$$|AC| = \sqrt{(3-1)^2+(6-2)^2+(4-3)^2} = \sqrt{21}$$

$$|BC| = \sqrt{(3-4)^2+(6-0)^2+(4-5)^2} = \sqrt{38}$$

$$|AB|^2+|AC|^2 = 17+21 = 38 = |BC|^2$$

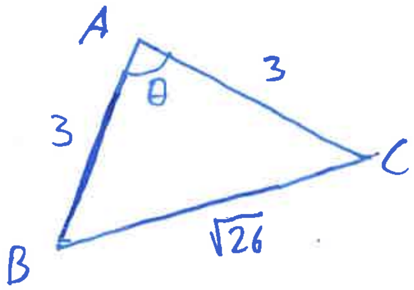
$\Rightarrow \Delta ABC$ er rettvinklet.

10.1.7

$$c = |AB| = \sqrt{(2-0)^2+(-1-1)^2+(-1+2)^2} = \sqrt{9} = 3$$

$$b = |AC| = \sqrt{(2-1)^2+(-1+3)^2+(-1-1)^2} = \sqrt{9} = 3$$

$$a = |BC| = \sqrt{(1-0)^2+(-3-1)^2+(1+2)^2} = \sqrt{26} \approx 5.1$$



$$a^2 = b^2+c^2-2bc\cdot\cos(\theta)$$

$$\theta = \cos^{-1}\left(\frac{b^2+c^2-a^2}{2\cdot b c}\right)$$

$$\theta = \cos^{-1}\left(\frac{9+9-26}{2\cdot 3\cdot 3}\right)$$

$$\theta = \cos^{-1}\left(-\frac{4}{9}\right) = 116.4^\circ$$

10.1.8

$A = (1,2,3),\ B = (1,3,4),\ C = (0,3,3)$

$$|AB| = \sqrt{(1-1)^2 + (3-2)^2 + (4-3)^2} = \sqrt{2}$$

$$|AC| = \sqrt{(1-0)^2 + (2-3)^2 + (3-3)^2} = \sqrt{2}$$

$$|BC| = \sqrt{(1-0)^2 + (3-3)^2 + (4-3)^2} = \sqrt{2}$$

$$|AB| = |AC| = |BC|$$

$\Rightarrow \triangle ABC$ er likesidet.

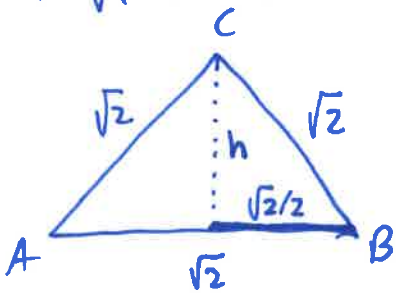
10.1.9

$A = (1,1,0),\ B = (1,0,1),\ C = (0,1,1)$

$$|AB| = \sqrt{(1-1)^2 + (1-0)^2 + (0-1)^2} = \sqrt{2}$$

$$|AC| = \sqrt{(1-0)^2 + (1-1)^2 + (0-1)^2} = \sqrt{2}$$

$$|BC| = \sqrt{(1-0)^2 + (0-1)^2 + (1-1)^2} = \sqrt{2}$$



$$h = \sqrt{(\sqrt{2})^2 - \left(\frac{\sqrt{2}}{2}\right)^2} = \sqrt{2 - \frac{1}{2}} = \sqrt{\frac{3}{2}}$$

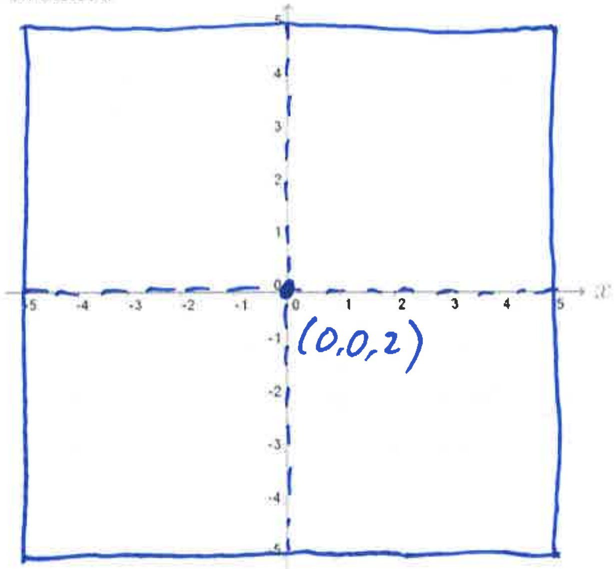
$$\text{Areal} = \frac{1}{2} \cdot \text{grunnlinje} \cdot \text{høyde}$$

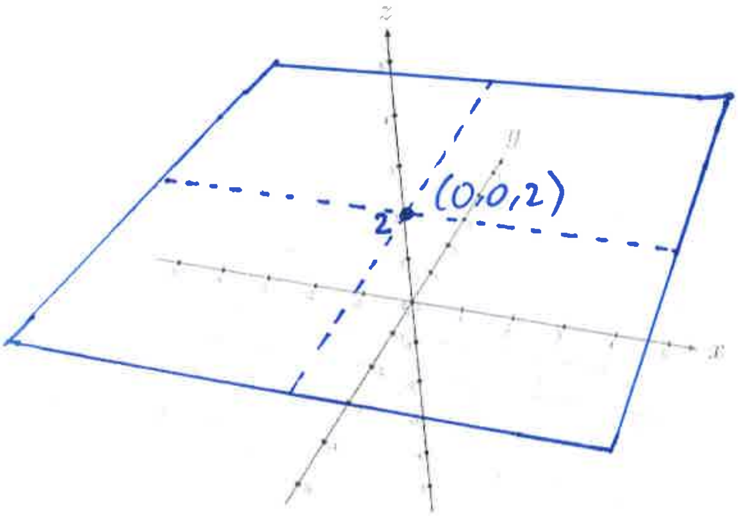
$$= \frac{1}{2} \cdot \sqrt{2} \cdot \sqrt{\frac{3}{2}} = \frac{\sqrt{3}}{2}$$

10.1.12

$z=2$ er et plan som står vinkelrett på z-aksen og går gjennom punktet $(0,0,2)$

ovenifra





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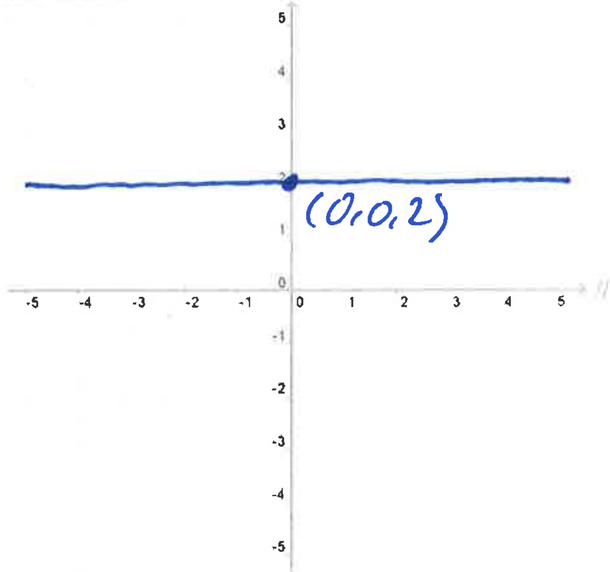

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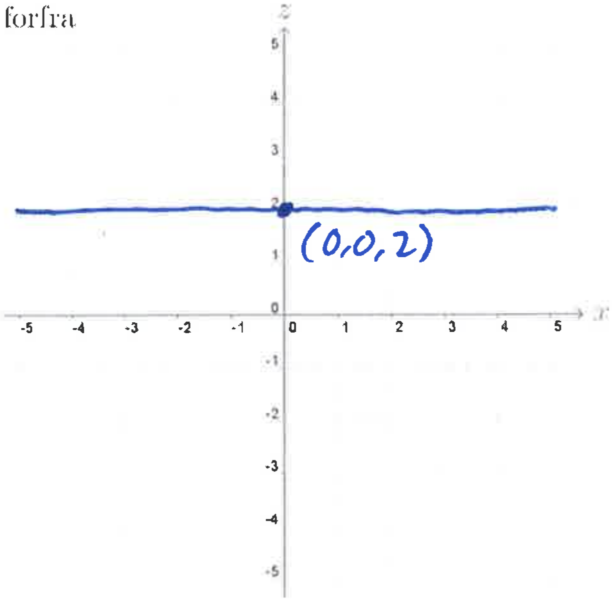
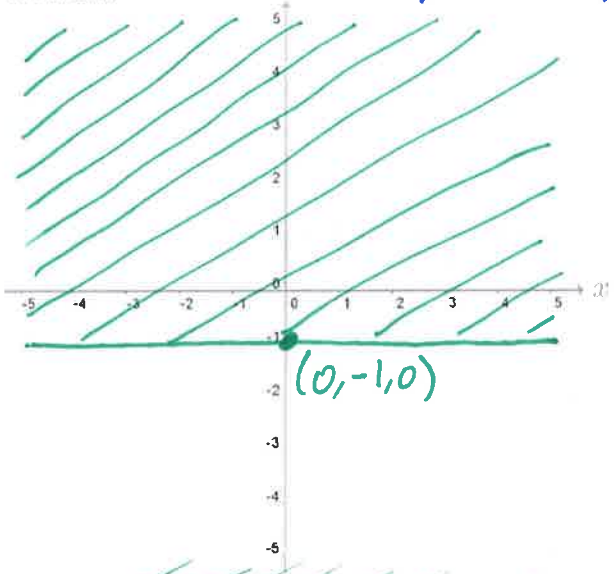


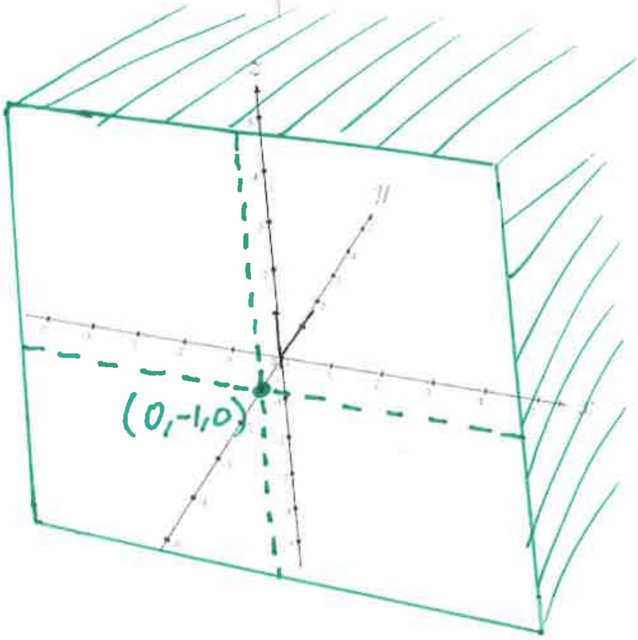
10.1.13

$y \geq -1$

Alle punkter på planet $y=-1$ (som står vinkelrett på y-aksen og går gjennom punktet $(0,-1,0)$) og alle punkter som er på samme side av planet som origo.

ovenifra





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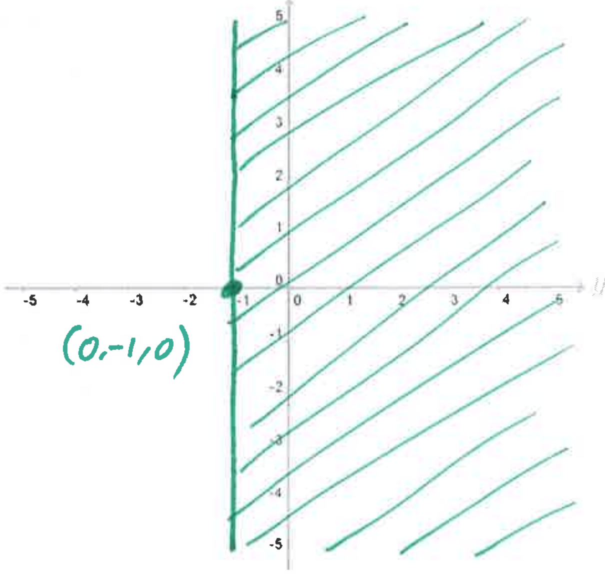

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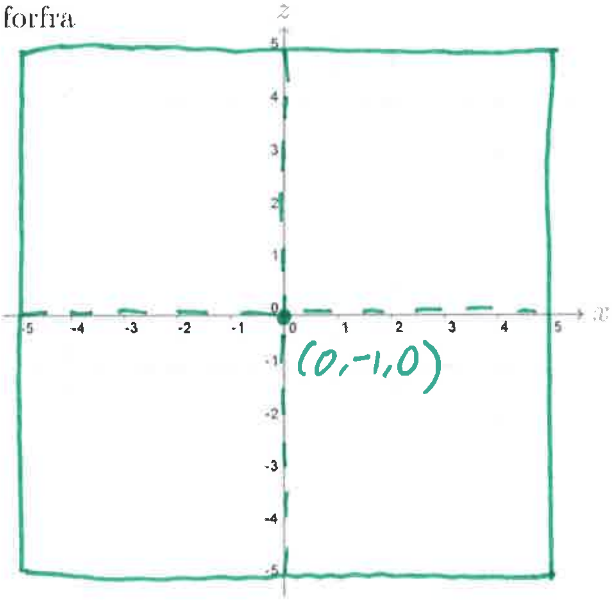
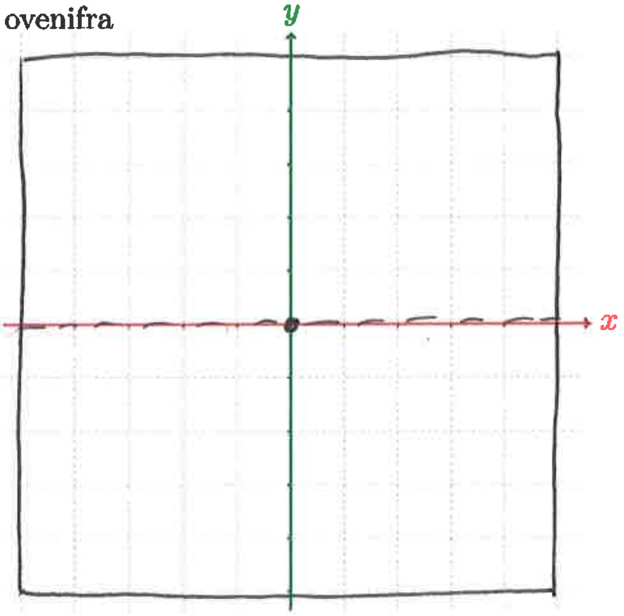
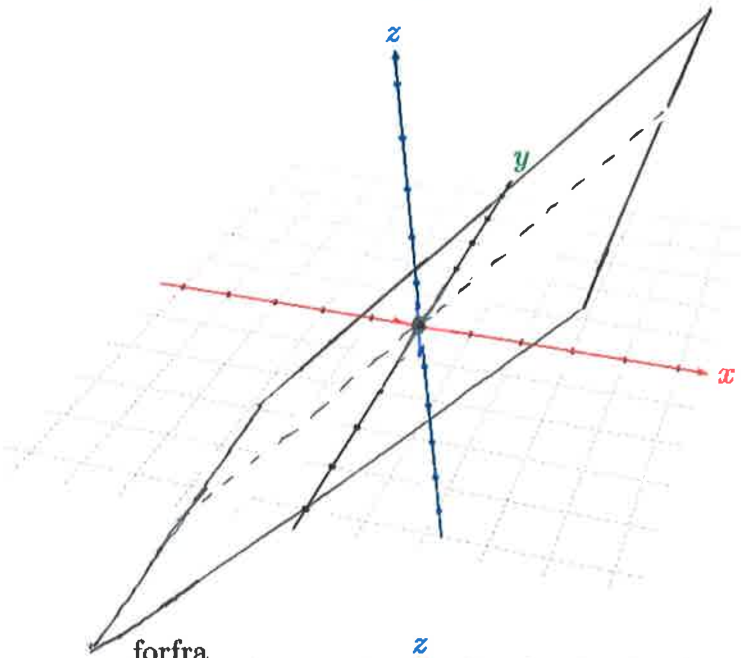


10.1.14 $z = x$ er planet som går gjennom origo, inneholder y-aksen og danner en 45° vinkel med x-aksen og z-aksen.

ovenifra





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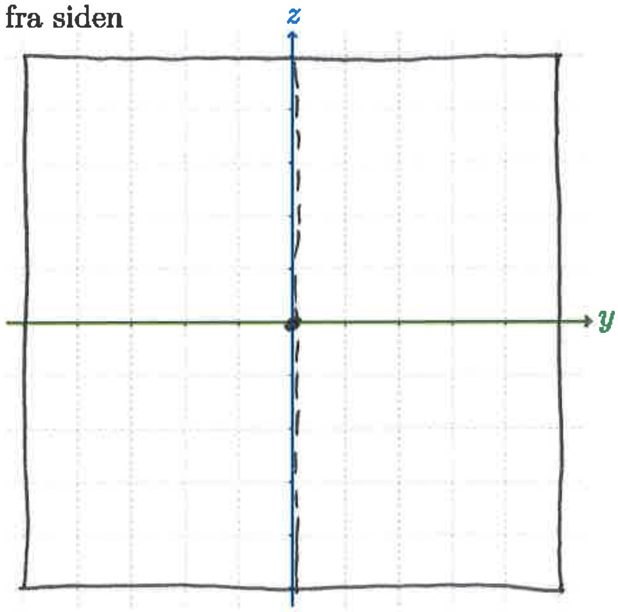

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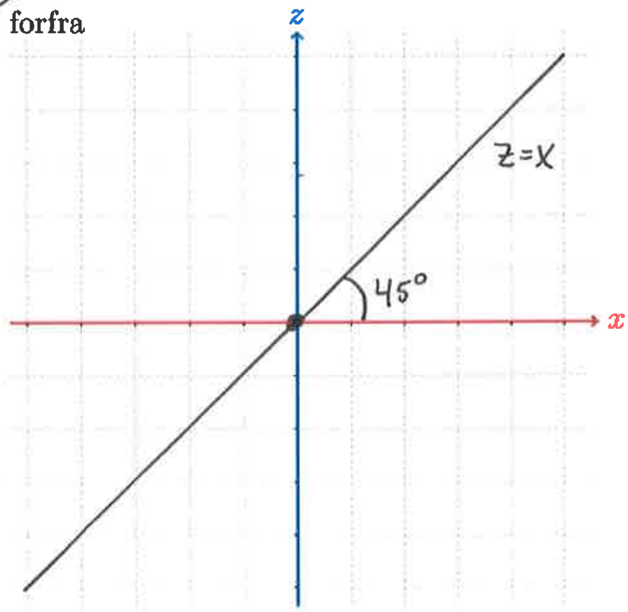
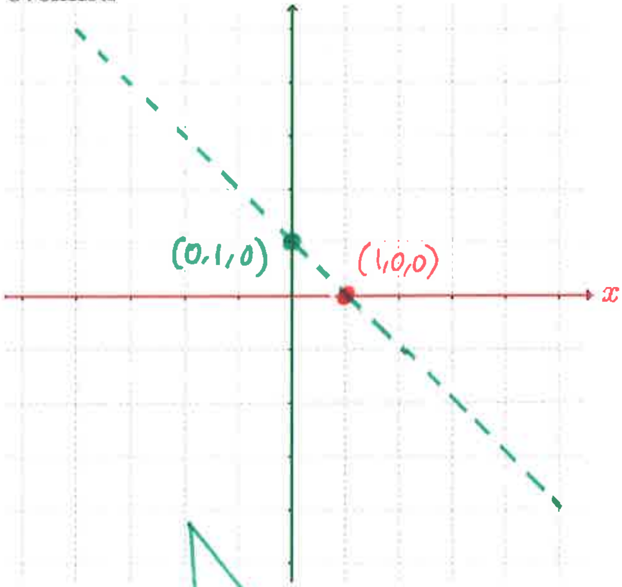
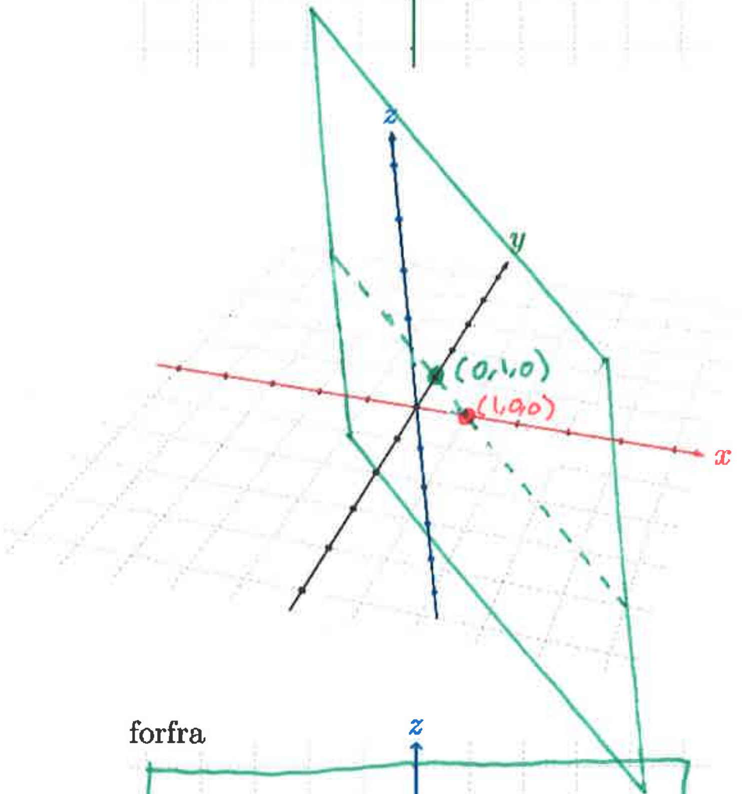


10.1.15 $x + y = 1 \Rightarrow y = -x + 1$ er et plan parallellt med z-aksen som går gjennom punktene $(0,1,0)$ og $(1,0,0)$

ovenifra





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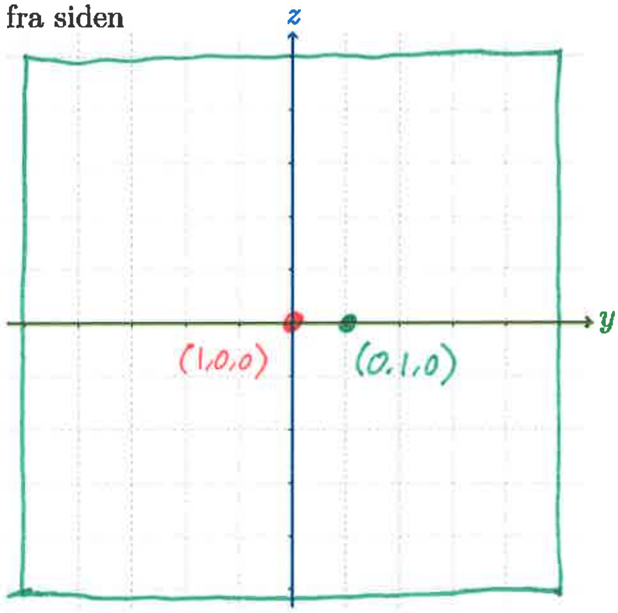

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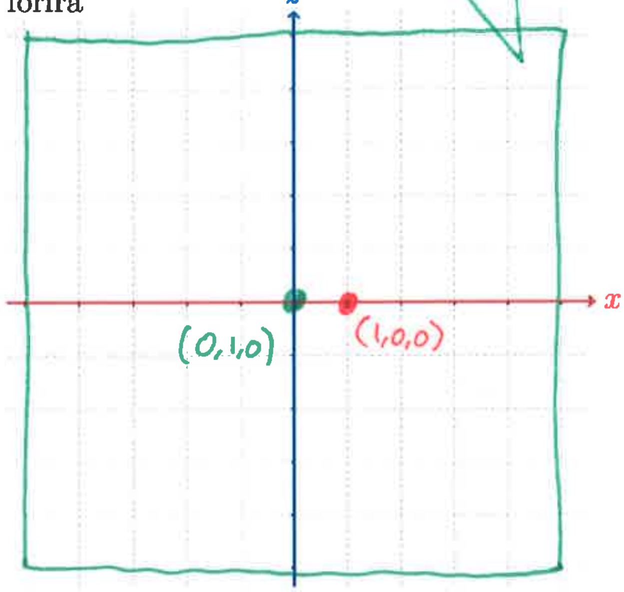
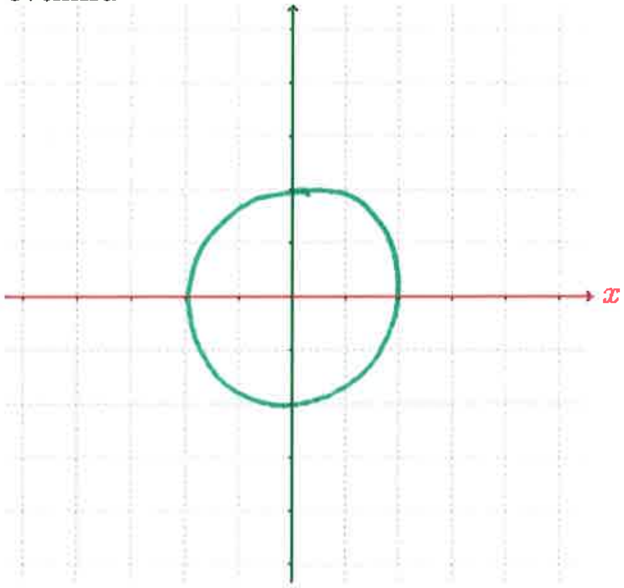
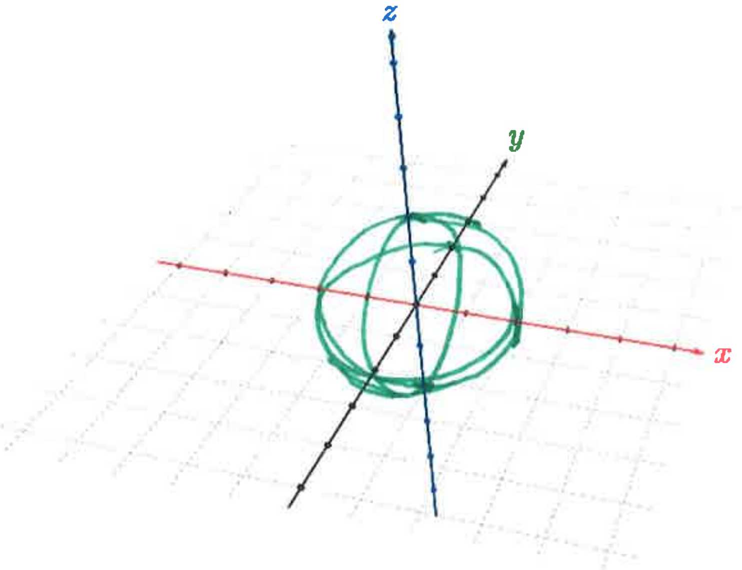


10.1.16 $x^2+y^2+z^2=4$ er en kule med senter i origo og radius lik 2.

ovenifra





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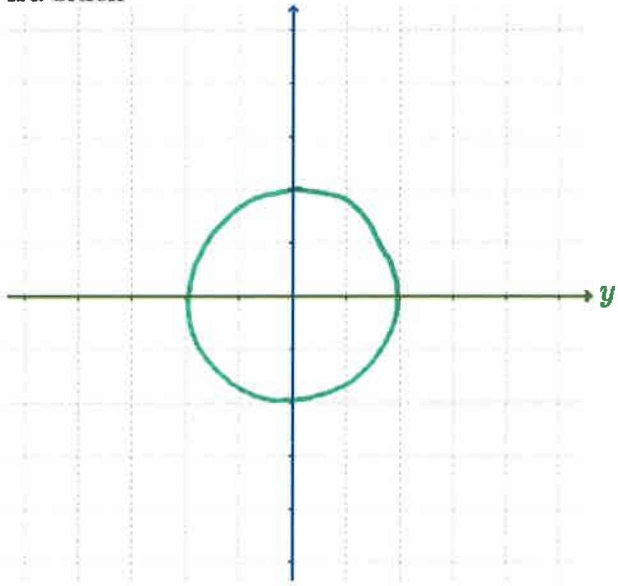

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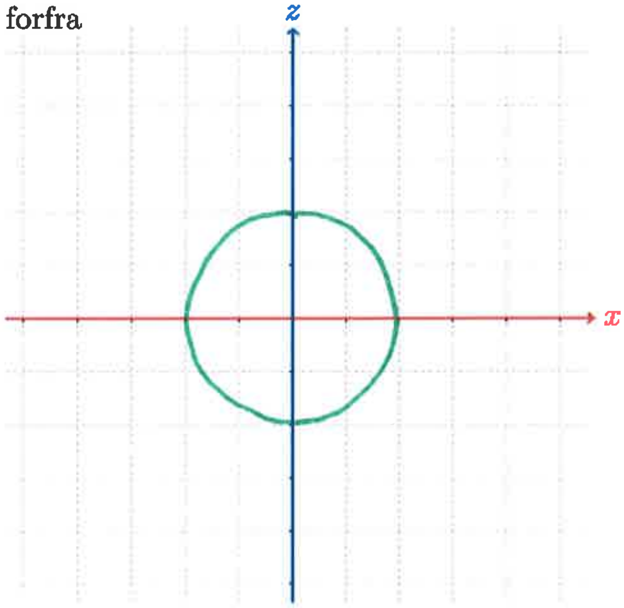
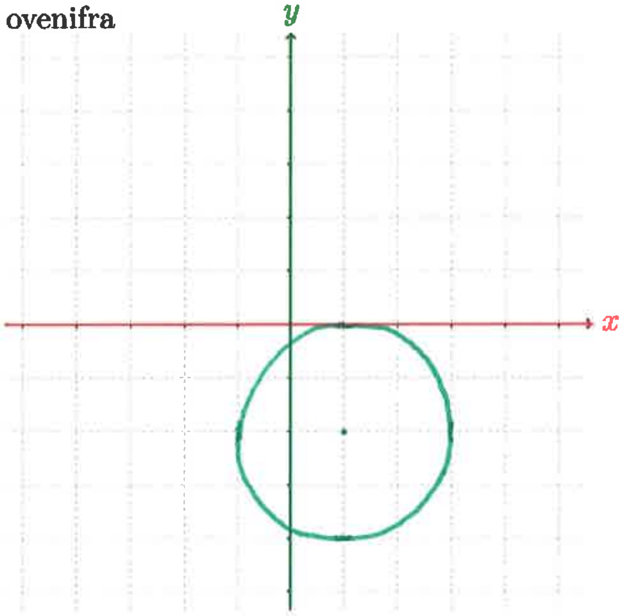
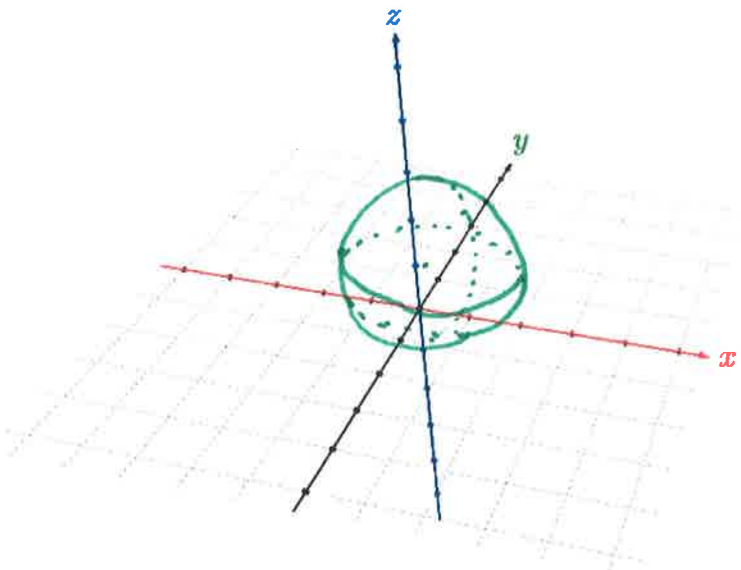
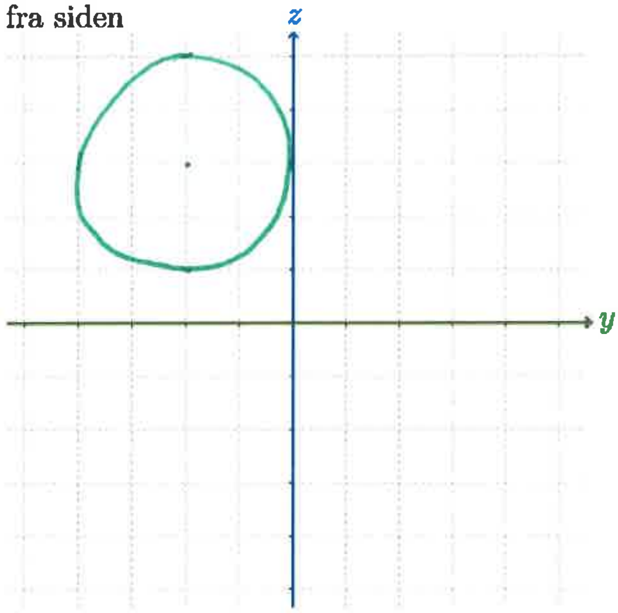
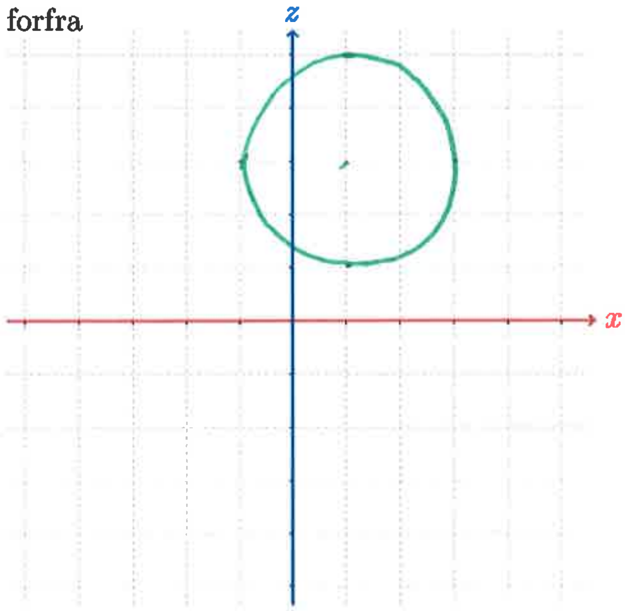


10.1.17 $(x-1)^2+(y+2)^2+(z-3)^2=4$ er en kule med senter i $(1,-2,3)$ og radius lik 2.









10.1.18

$$x^2+y^2+z^2=2z$$

$$x^2+y^2+z^2-2z=0$$

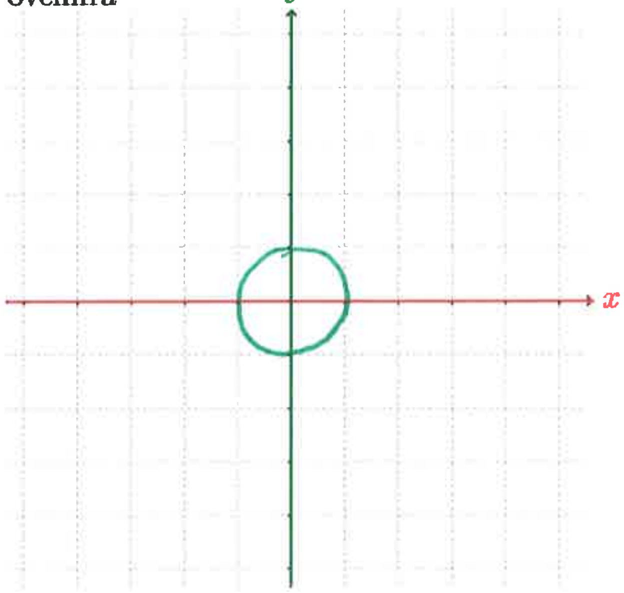
$$x^2+y^2+\left(z+\frac{-2}{2}\right)^2-\frac{(-2)^2}{4}=0$$

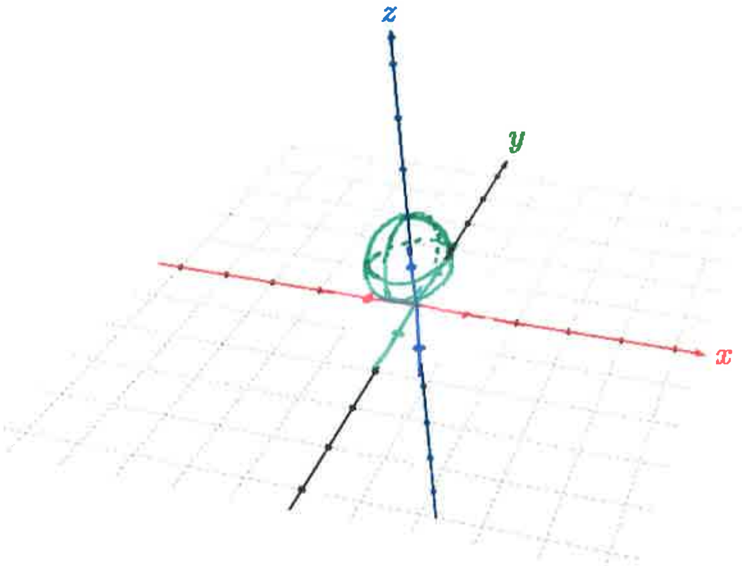
$$x^2+y^2+(z-1)^2=1$$

Kule med senter i $(0,0,1)$

og radius $=1$

ovenifra





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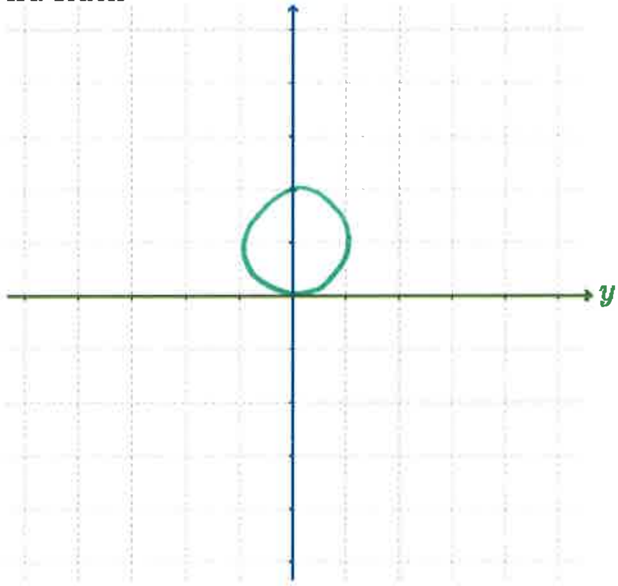

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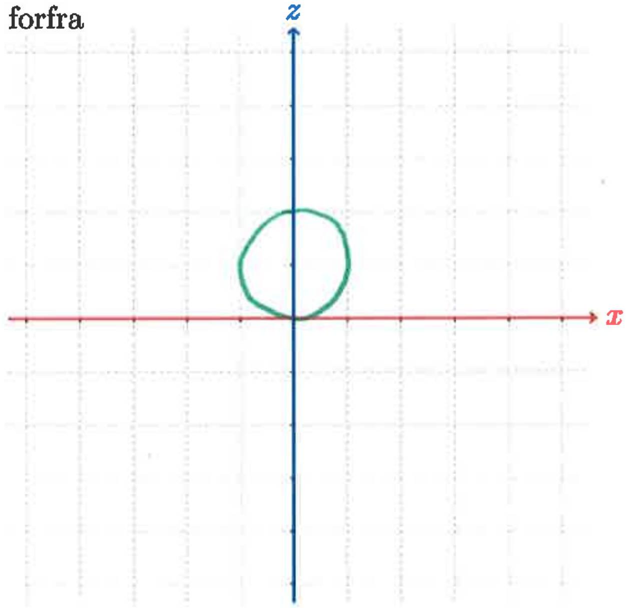
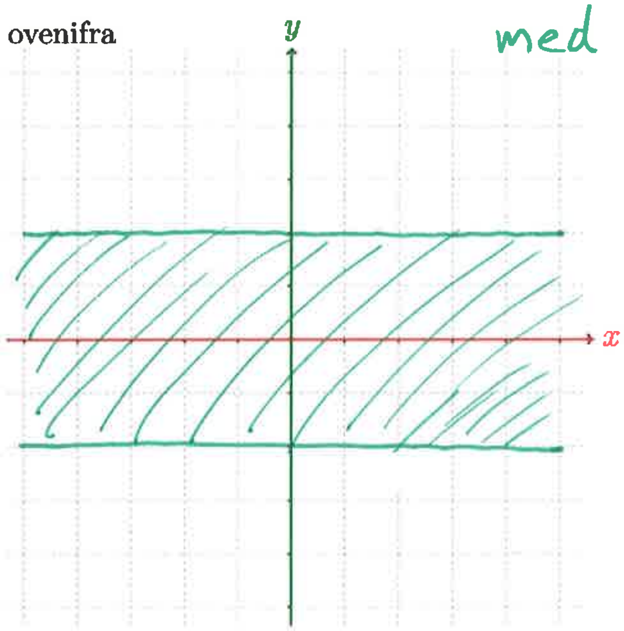
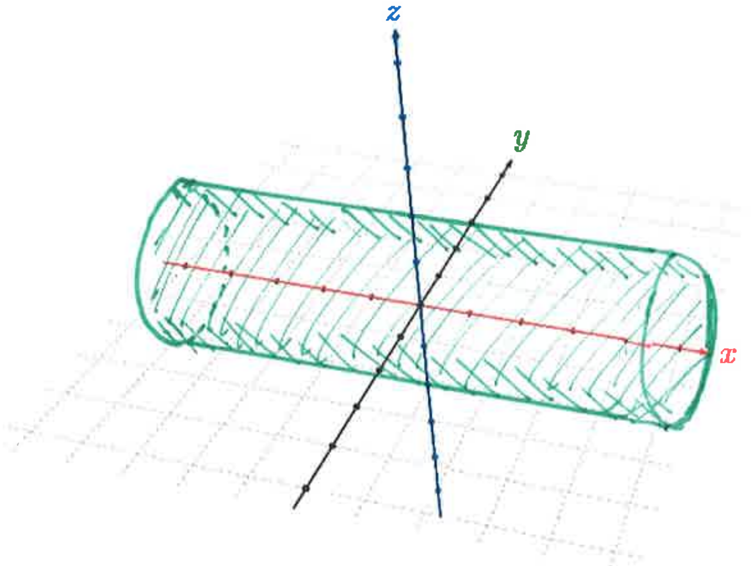


10.1.19 $y^2+z^2 \le 4$ Alle punkter på overflaten til, og inni sylinderen som er parallell med x-aksen, har akse langs x-aksen og radius lik 2.

ovenifra





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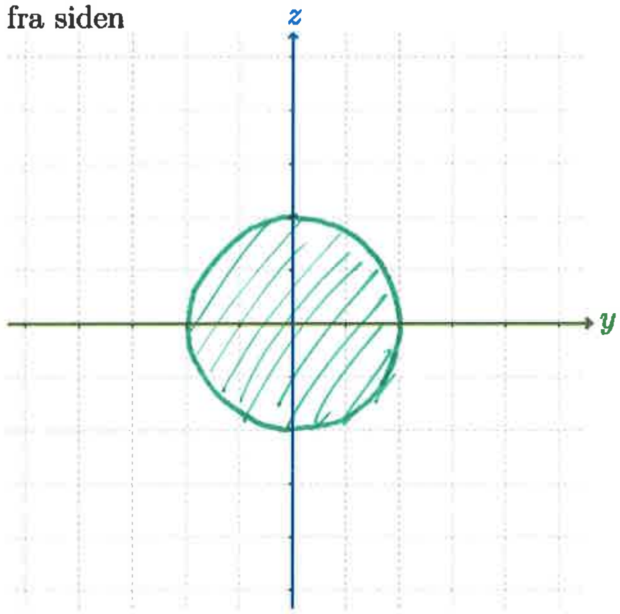

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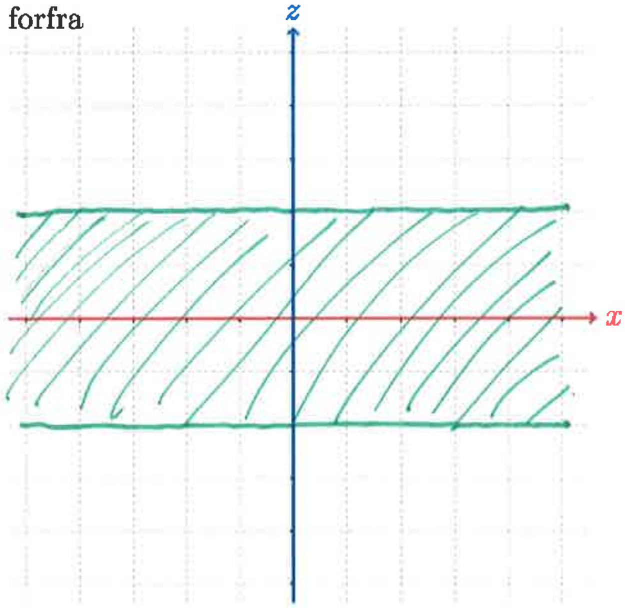
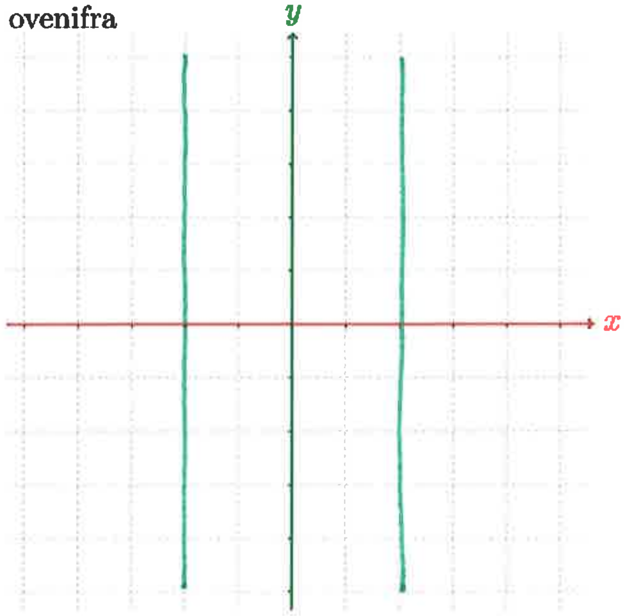
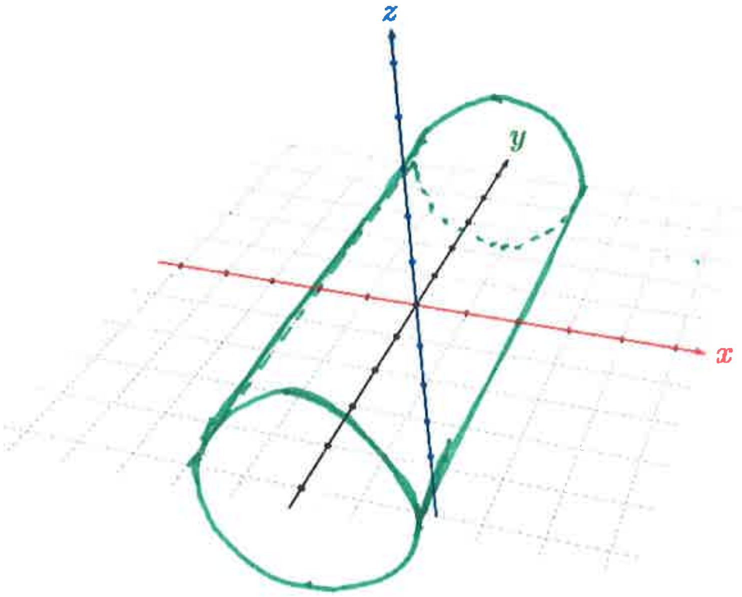


<u>10.1.20</u> $x^2+z^2=4$ sylinder med radius 2 med akse langs y-aksen

ovenifra





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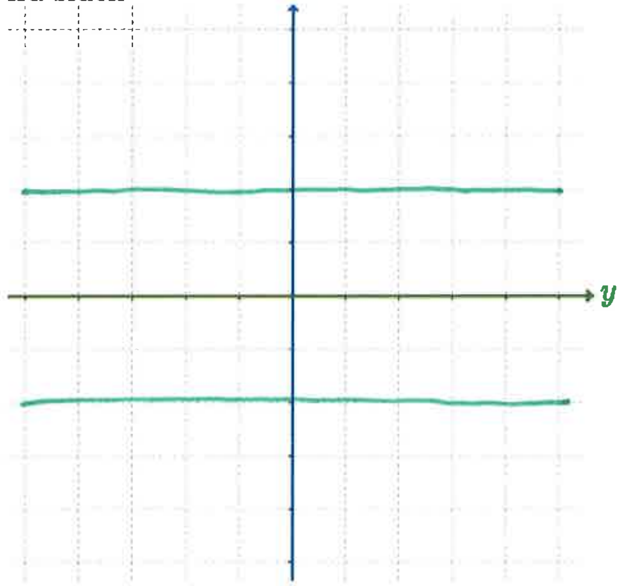

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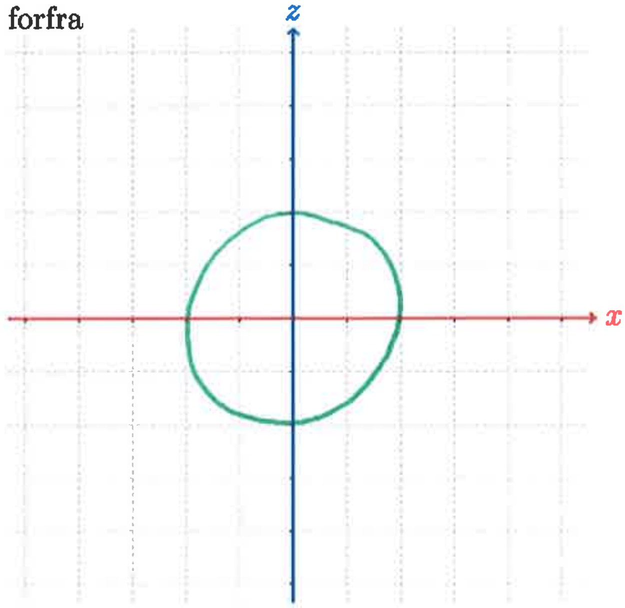
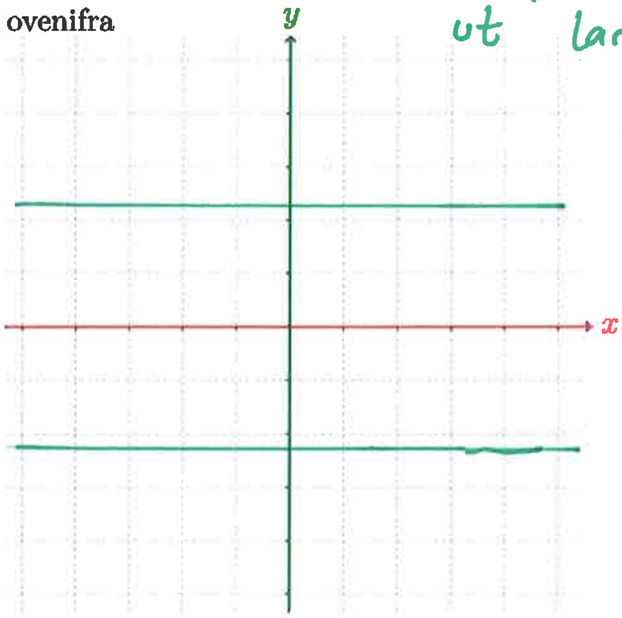
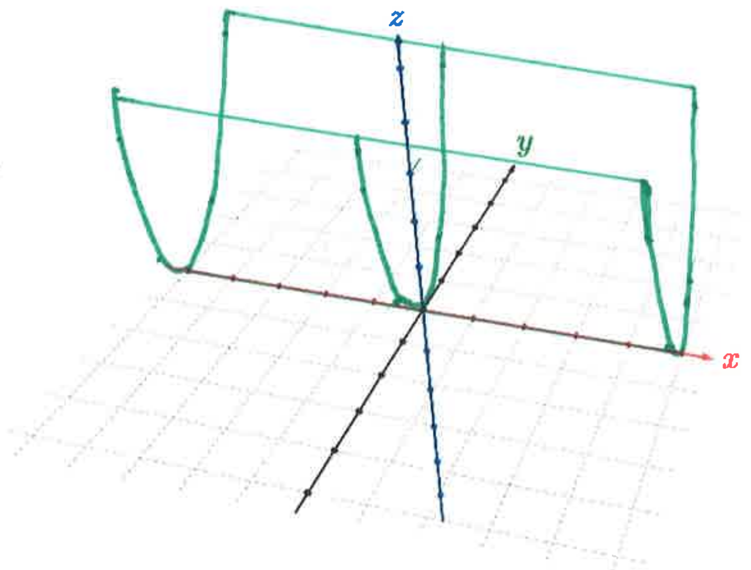


10.1.21 $z = y^2$ er en såkalt parabolisk sylinder; en parabelform som strekker seg ut langs x-aksen med toppunktet på x-aksen.

ovenifra





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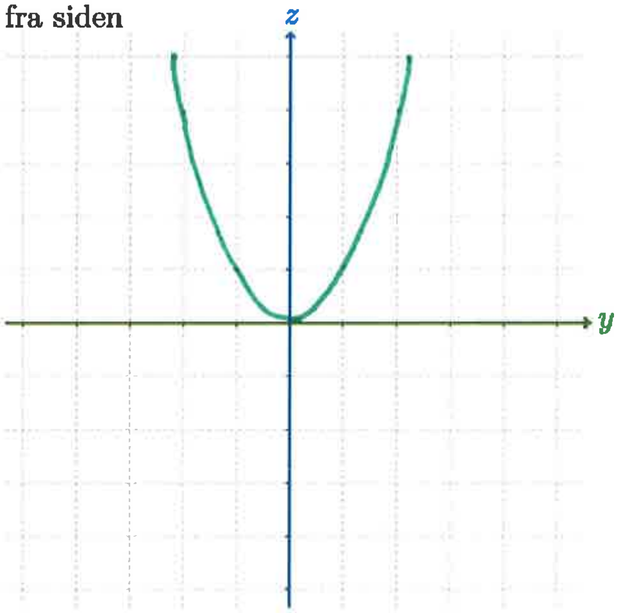

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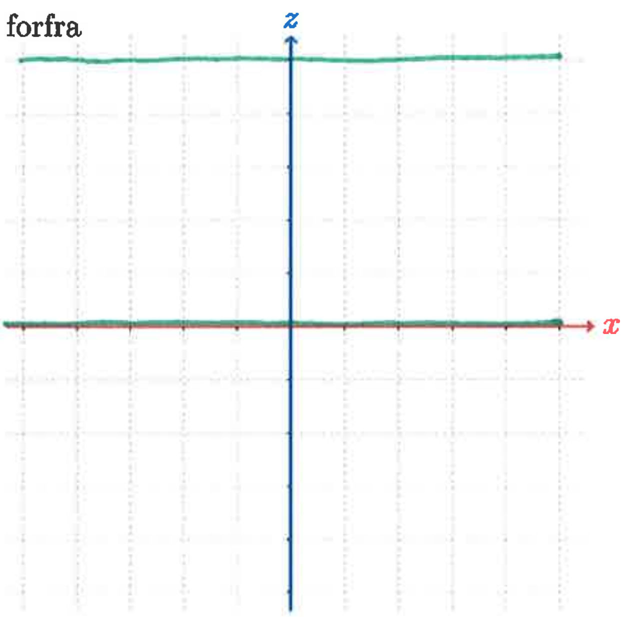
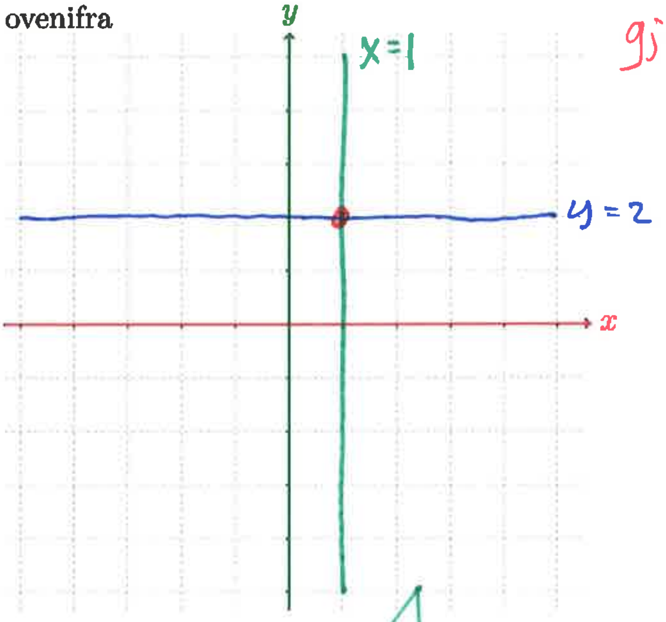
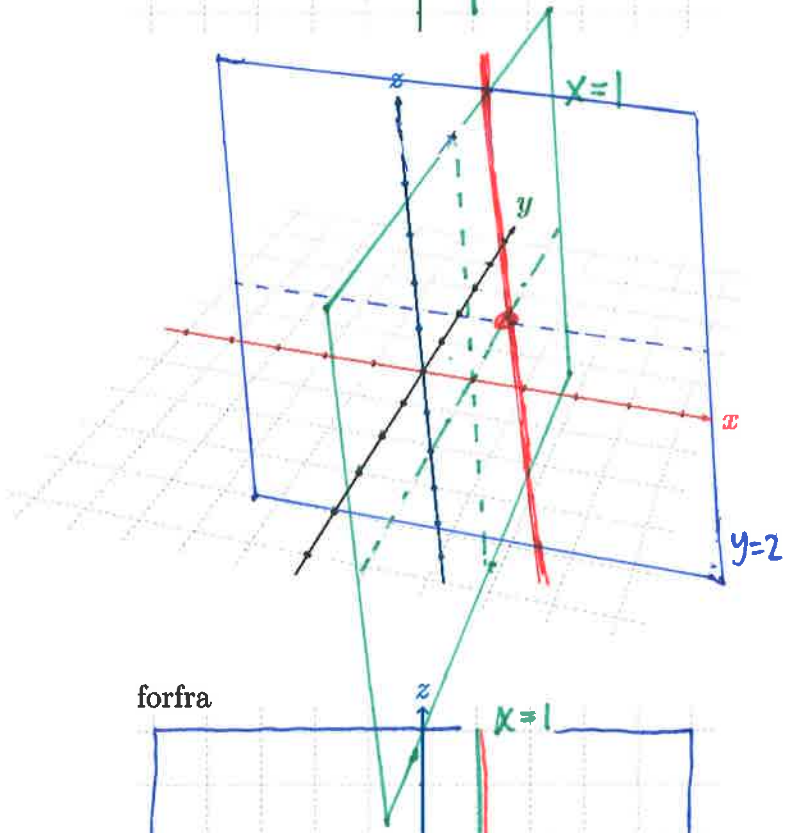


10.1.24

$$\begin{cases} x = 1 \\ y = 2 \end{cases}$$ = Skjæring mellom to plan i en linje parallell med z-aksen som går gjennom punktet (1,2,0)

ovenifra





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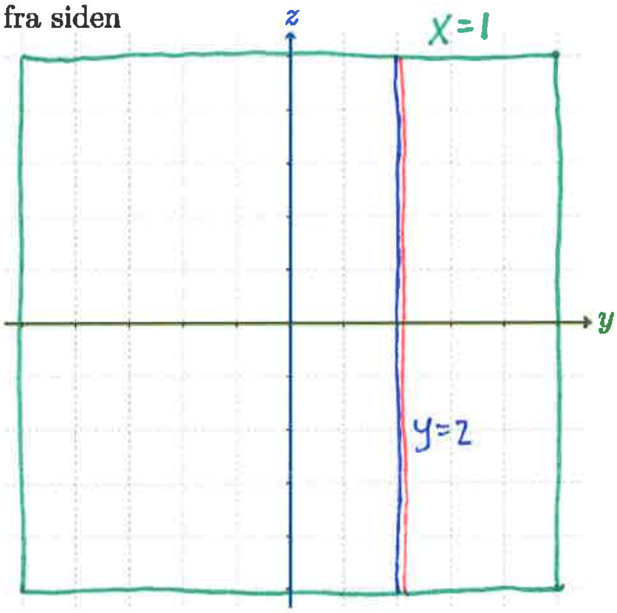

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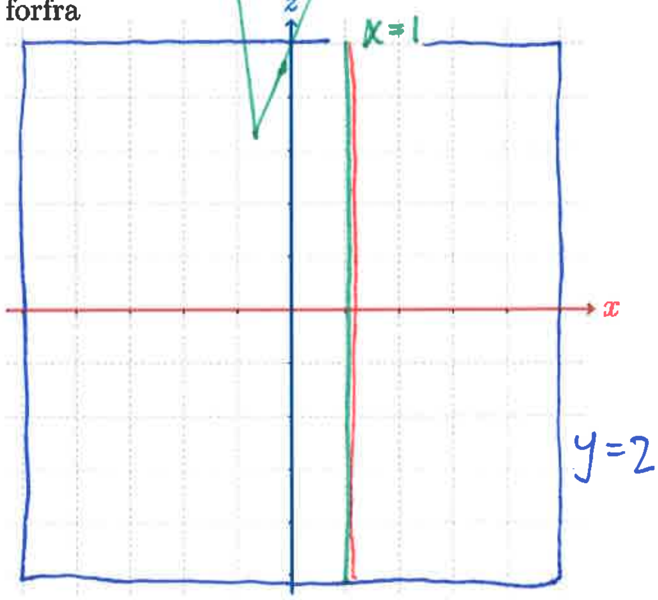


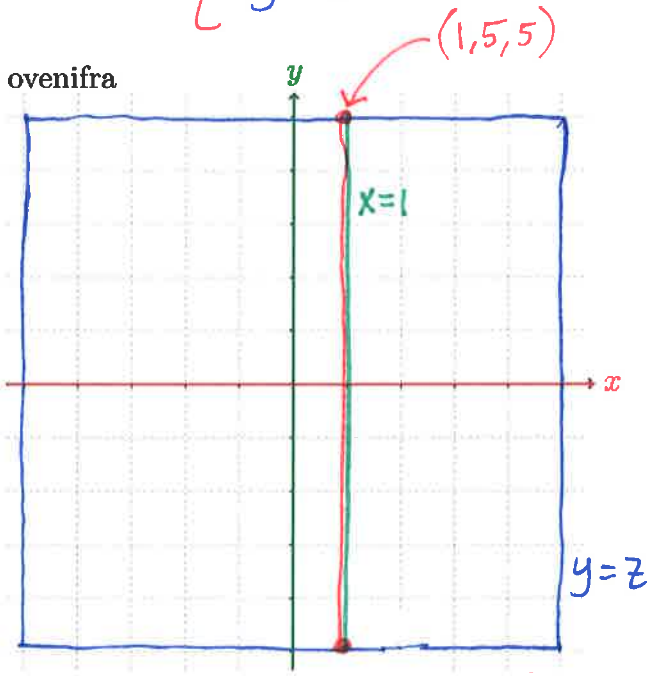
10.1.25

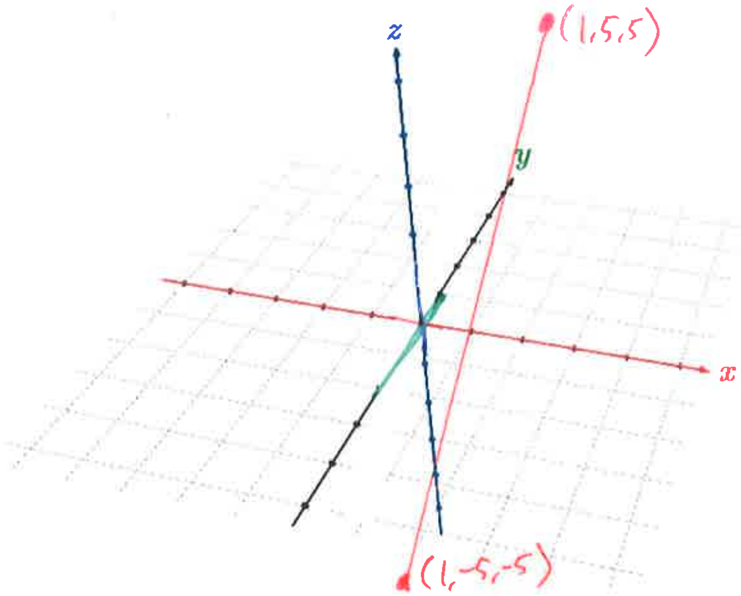
$$\begin{cases} x = 1 \\ y = z \end{cases}$$

Skjæring mellom to plan som gir en rett linje som går gjennom

$(1,5,5)$, $(1,0,0)$, $(1,-5,-5)$

ovenifra





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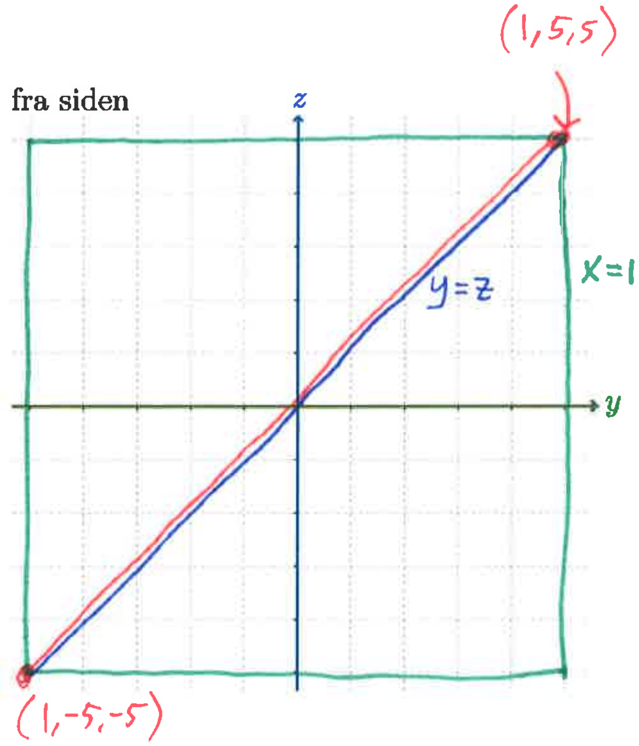

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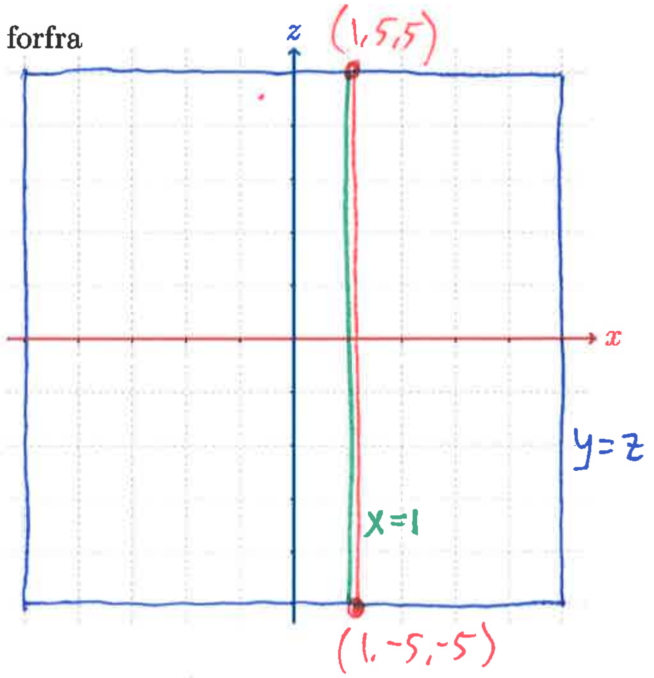
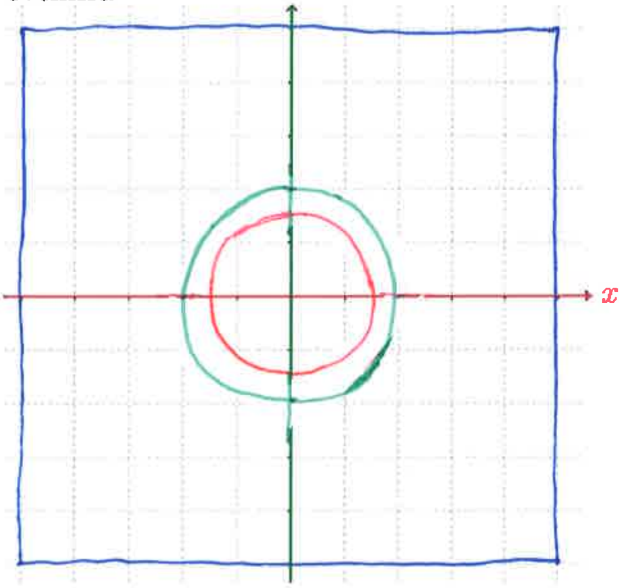


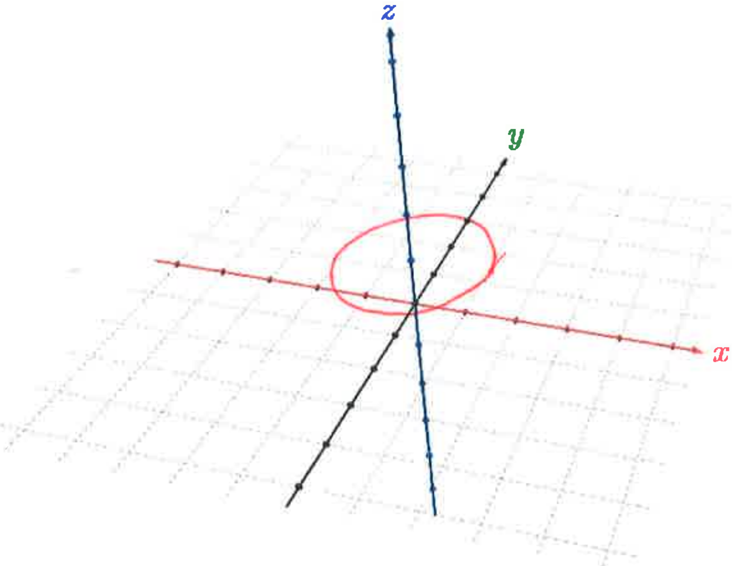
10.1.26

$$\begin{cases} x^2+y^2+z^2=4 & \text{kule med senter i origo og radius 2} \\ z=1 & \text{plan parallellt med xy-planet gjennom } (0,0,1) \end{cases}$$

Skjæring mellom kule og plan gir en sirkel som er parallell med xy-planet, senter i $(0,0,1)$ og radius lik $\sqrt{3}$.

ovenifra





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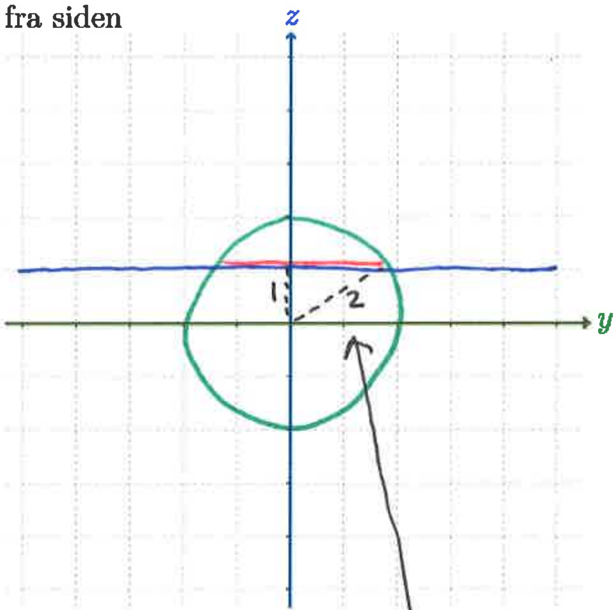
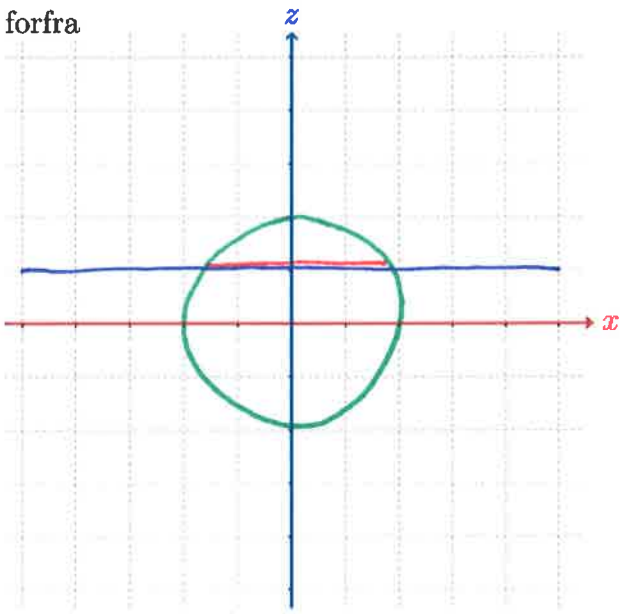


$x^2+1^2=2^2$

$x=\sqrt{3}$

$\Rightarrow$ sirkel har radius $\sqrt{3}$.

forfra

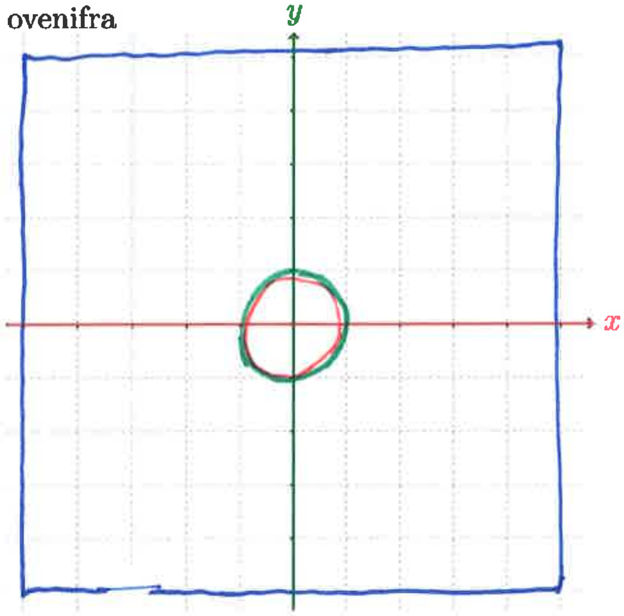


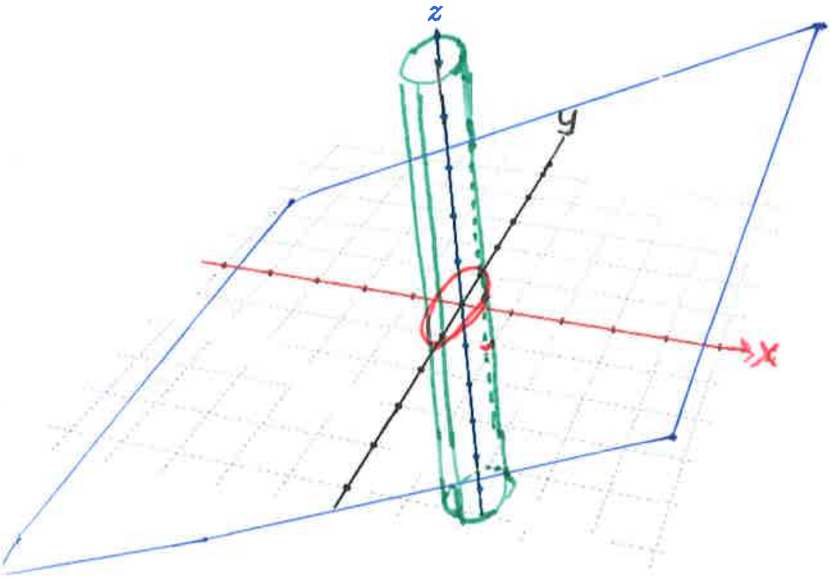
10.1.29

$$\begin{cases} x^2+y^2=1 \\ z=x \end{cases}$$

Skjæring mellom sylinder og plan gir en sirkel her med senter i origo, 45° rotert ift xy-planet, en diameter på y-aksen og radius lik $\sqrt{2}$

ovenifra





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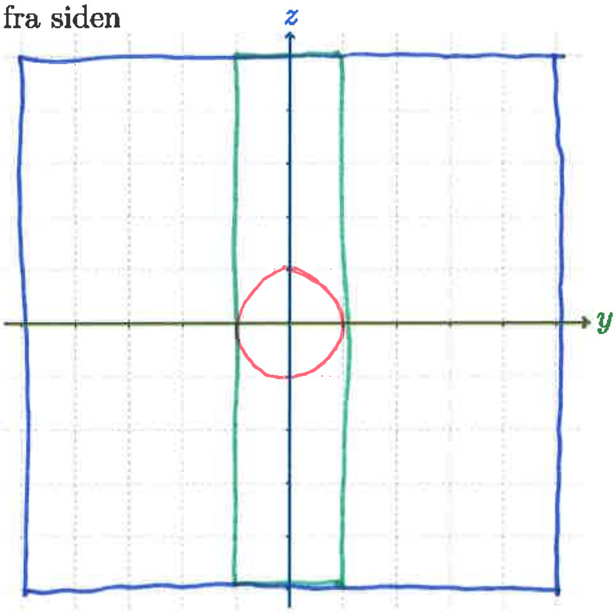

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