10.2.1

$A=(-1,2)$, $B=(2,0)$, $C=(1,-3)$, $D=(0,4)$

a) $\vec{AB} = (2-(-1))\vec{i} + (0-2)\vec{j} = 3\vec{i} - 2\vec{j}$

b) $\vec{BA} = -\vec{AB} = -3\vec{i} + 2\vec{j}$

c) $\vec{AC} = (1-(-1))\vec{i} + (-3-2)\vec{j} = 2\vec{i} - 5\vec{j}$

d) $\vec{BD} = (0-2)\vec{i} + (4-0)\vec{j} = -2\vec{i} + 4\vec{j}$

e) $\vec{DA} = (-1-0)\vec{i} + (2-4)\vec{j} = -\vec{i} - 2\vec{j}$

f) $\vec{BC} = (1-2)\vec{i} + (-3-0)\vec{j} = -\vec{i} - 3\vec{j}$

$\vec{AB} - \vec{BC} = 3\vec{i} - 2\vec{j} - (-\vec{i} - 3\vec{j})$

$= 4\vec{i} + \vec{j}$

g) $\vec{CD} = (0-1)\vec{i} + (4-(-3))\vec{j} = -\vec{i} + 7\vec{j}$

$\vec{AC} - 2\vec{AB} + 3\vec{CD} = \vec{AC} + 2\vec{BA} + 3\vec{CD}$

$= (2 + 2\cdot(-3) + 3\cdot(-1))\vec{i} + (-5 + 2\cdot 2 + 3\cdot 7)\vec{j}$

$= -7\vec{i} + 20\vec{j}$

h) $\vec{AD} = -\vec{DA} = \vec{i} + 2\vec{j}$

$(\vec{AB} + \vec{AC} + \vec{AD})/3$

$= \left((3+2+1)\vec{i} + (-2-5+2)\vec{j}\right)/3$

$= \left(6\vec{i} - 5\vec{j}\right)/3 = 2\vec{i} - \frac{5}{3}\vec{j}$

10.2.2

$\vec{u} = \vec{i} - \vec{j}$, $\vec{v} = \vec{j} + 2\vec{k}$

a) $\vec{u} + \vec{v} = \vec{i} + 2\vec{k}$

$\vec{u} - \vec{v} = \vec{i} - 2\vec{j} - 2\vec{k}$

$2\vec{u} - 3\vec{v} = 2\vec{i} - 2\vec{j} - (3\vec{j} + 6\vec{k})$

$= 2\vec{i} - 5\vec{j} - 6\vec{k}$

b) $|\vec{u}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$|\vec{v}| = \sqrt{1^2 + 2^2} = \sqrt{5}$

c) $\hat{u} = \frac{1}{|\vec{u}|}\cdot\vec{u} = \frac{1}{\sqrt{2}}(\vec{i} - \vec{j}) = \frac{\sqrt{2}}{2}\vec{i} - \frac{\sqrt{2}}{2}\vec{j}$

$\hat{v} = \frac{1}{|\vec{v}|}\cdot\vec{v} = \frac{1}{\sqrt{5}}(\vec{j} + 2\vec{k}) = \frac{\sqrt{5}}{5}\vec{j} + \frac{2\sqrt{5}}{5}\vec{k}$

d) $\vec{u}\cdot\vec{v} = 1\cdot 0 + (-1)\cdot 1 + 0\cdot 2 = -1$

e) $\theta = \cos^{-1}\left(\frac{\vec{u}\cdot\vec{v}}{|\vec{u}|\cdot|\vec{v}|}\right) = \cos^{-1}\left(\frac{-1}{\sqrt{10}}\right) = 108.4^\circ$

f) $s = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|} = |\vec{u}|\cdot\cos\theta = \sqrt{2}\cdot\cos(108.4^\circ) = -\frac{1}{\sqrt{5}}$

g) $\vec{v}_{\vec{u}} = \frac{\vec{v}\cdot\vec{u}}{|\vec{u}|}\cdot\hat{u} = \frac{-1}{\sqrt{2}}\cdot\left(\frac{\sqrt{2}}{2}\vec{i} - \frac{\sqrt{2}}{2}\vec{j}\right) = -\frac{1}{2}\vec{i} + \frac{1}{2}\vec{j}$

10.2.3

$\vec{u} = 3\vec{i} + 4\vec{j} - 5\vec{k}$, $\vec{v} = 3\vec{i} - 4\vec{j} - 5\vec{k}$

a) $\vec{u} + \vec{v} = 6\vec{i} - 10\vec{k}$, $\vec{u} - \vec{v} = 8\vec{j}$,

$2\vec{u} - 3\vec{v} = (6-9)\vec{i} + (8+12)\vec{j} + (-10+15)\vec{k} = -3\vec{i} + 20\vec{j} + 5\vec{k}$

b) $|\vec{u}| = \sqrt{9+16+25} = 5\sqrt{2}$, $|\vec{v}| = \sqrt{9+16+25} = 5\sqrt{2}$

c) $\hat{u} = \frac{1}{|\vec{u}|}\cdot\vec{u} = \frac{1}{5\sqrt{2}}\cdot(3\vec{i} + 4\vec{j} - 5\vec{k}) = \frac{3\sqrt{2}}{10}\vec{i} + \frac{2\sqrt{2}}{5}\vec{j} - \frac{\sqrt{2}}{2}\vec{k}$

$\hat{v} = \frac{1}{|\vec{v}|}\cdot\vec{v} = \frac{1}{5\sqrt{2}}\cdot(3\vec{i} - 4\vec{j} - 5\vec{k}) = \frac{3\sqrt{2}}{10}\vec{i} - \frac{2\sqrt{2}}{5}\vec{j} - \frac{\sqrt{2}}{2}\vec{k}$

d) $\vec{u}\cdot\vec{v} = 3\cdot 3 + 4\cdot(-4) + (-5)\cdot(-5) = 9 - 16 + 25 = 18$

e) $\theta = \cos^{-1}\left(\frac{\vec{u}\cdot\vec{v}}{|\vec{u}|\cdot|\vec{v}|}\right) = \cos^{-1}\left(\frac{18}{50}\right) = \cos^{-1}\left(\frac{9}{25}\right) = 68.9^\circ$

f) $s = |\vec{u}|\cdot\cos(68.9^\circ) = 5\sqrt{2}\cdot\frac{9}{25} = \frac{9\sqrt{2}}{5}$

g) $\vec{v}_{\vec{u}} = \frac{\vec{v}\cdot\vec{u}}{|\vec{u}|}\cdot\hat{u} = \frac{18}{5\sqrt{2}}\cdot\left(\frac{3\sqrt{2}}{10}\vec{i} + \frac{2\sqrt{2}}{5}\vec{j} - \frac{\sqrt{2}}{2}\vec{k}\right)$

$= \frac{27}{25}\vec{i} + \frac{36}{25}\vec{j} - \frac{9}{5}\vec{k}$

10.2.4

$A=(-1,1)$, $B=(2,5)$, $C=(10,-1)$

$\vec{AB} = 3\vec{i} + 4\vec{j}$, $\vec{AC} = 11\vec{i} - 2\vec{j}$, $\vec{BC} = 8\vec{i} - 6\vec{j}$

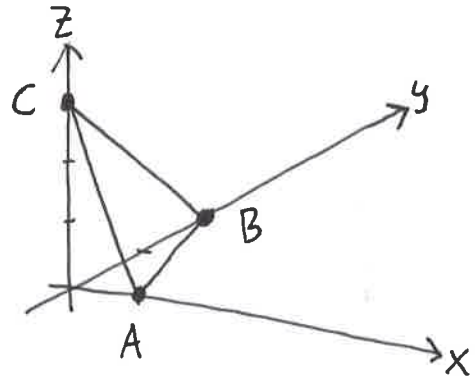
$|\vec{AB}| = 5$, $|\vec{AC}| = 5\sqrt{5} = 11.18$, $|\vec{BC}| = 10$

Hvis $\triangle ABC$ er retvinklet må AC være hypotenusen.

$|\vec{AB}|^2 + |\vec{BC}|^2 = 25 + 100 = 125 = (5\sqrt{5})^2 = |\vec{AC}|^2 \Rightarrow$ retvinklet.

10.2.18

$A=(1,0,0),\ B=(0,2,0),\ C=(0,0,3)$



$$\angle ABC = \cos^{-1}\left(\frac{\vec{BA}\cdot\vec{BC}}{|\vec{BA}|\cdot|\vec{BC}|}\right)$$

$$= \cos^{-1}\left(\frac{(\vec{i}-2\vec{j})\cdot(-2\vec{j}+3\vec{k})}{\sqrt{1+4}\cdot\sqrt{4+9}}\right)$$

$$= \cos^{-1}\left(\frac{-1\cdot 0+2\cdot 2+0\cdot 3}{\sqrt{5}\cdot\sqrt{13}}\right)$$

$$= \cos^{-1}\left(\frac{4}{\sqrt{65}}\right) = 60{,}26^\circ$$

$$\angle BCA = \cdots = \cos^{-1}\left(\frac{9}{\sqrt{10}\cdot\sqrt{13}}\right) = 37{,}87^\circ$$

$$\angle CAB = \cdots = \cos^{-1}\left(\frac{1}{\sqrt{10}\cdot\sqrt{5}}\right) = 81{,}87^\circ$$