3 (a) (i) $x^2+y^2-8x-4y=-11$

$$x^2-8x+y^2-4y=-11$$

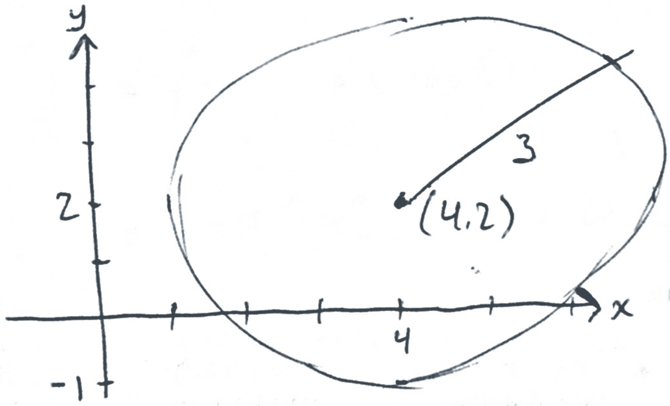
$$\left(x+\frac{-8}{2\cdot1}\right)^2-\frac{(-8)^2}{4\cdot1}+\left(y+\frac{-4}{2\cdot1}\right)-\frac{(-4)^2}{4\cdot1}=-11$$

$$(x-4)^2+(y-2)^2=-11+\frac{64}{4}+\frac{16}{4}$$

$$(x-4)^2+(y-2)^2=3^2$$

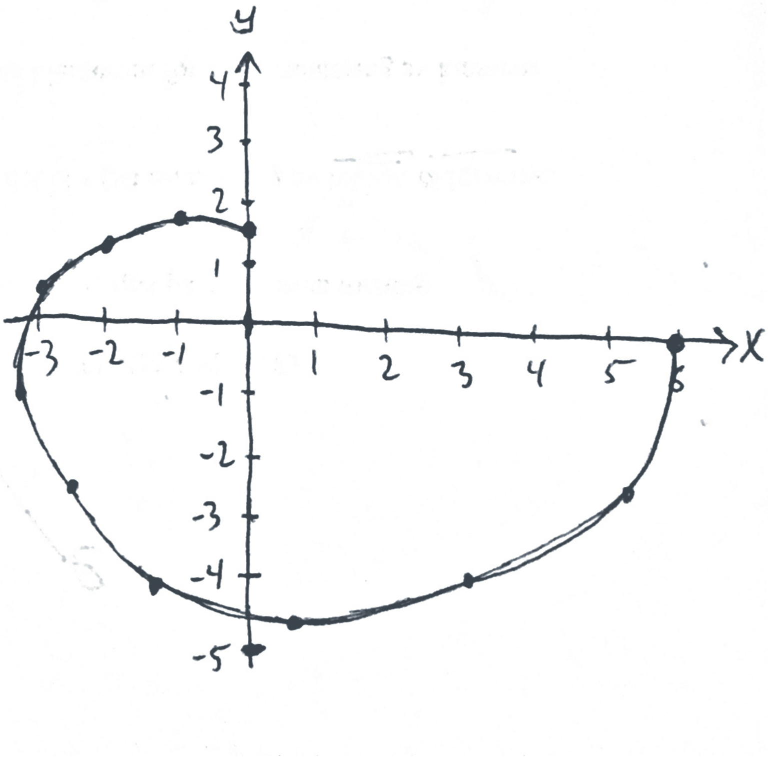
Sirkel med senter i $(4,2)$ og radius 3.

(ii)



(b) (i)

t	x	y
1.57	0	1.57
2.04	-0.93	1.82
2.51	-2.03	1.48
2.98	-2.95	0.47
3.45	-3.29	-1.07
3.93	-2.78	-2.77
4.39	-1.36	-4.18
4.87	0.76	-4.81
5.34	3.14	-4.32
5.81	5.18	-2.64
6.28	6.28	0



3 (b) (ii) $\begin{cases} x = t \cdot \cos(t) \\ y = t \cdot \sin(t) \end{cases}$ $(\pi/2 < t < 2\pi)$

$$\frac{dx}{dt} = \cos(t) - t \cdot \sin(t)$$

$$\frac{dy}{dt} = \sin(t) + t \cdot \cos(t)$$

$$\left(\frac{dx}{dt}\right)^2 = \left(\cos(t) - t \cdot \sin(t)\right)^2$$

$$= \cos(t)^2 - 2t \cdot \sin(t) \cdot \cos(t) + t^2 \cdot \sin(t)^2$$

$$\left(\frac{dy}{dt}\right)^2 = \left(\sin(t) + t \cdot \cos(t)\right)^2$$

$$= \sin(t)^2 + 2t \cdot \sin(t) \cdot \cos(t) + t^2 \cdot \cos(t)^2$$

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \underbrace{\cos(t)^2 + \sin(t)^2}_{1} + t^2 \cdot \left(\underbrace{\sin(t)^2 + \cos(t)^2}_{1}\right)$$

$$= t^2 + 1$$

$$S = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_{\pi/2}^{2\pi} \sqrt{t^2+1}\, dt$$

3 (c) (i) $(x,y,z) = (1,2,-8)$

$$R = \sqrt{x^2+y^2+z^2} = \sqrt{1+4+64} = \sqrt{69} \approx 8.31$$

$$\phi = \cos^{-1}(z/R) = \cos^{-1}(-8/\sqrt{69}) \approx 2.87$$

$$\theta = \operatorname{atan2}(y,x) = \operatorname{atan2}(2,1) = \tan^{-1}\left(\frac{2}{1}\right) \approx 1.11$$

Kulekoordinater:

$$[R,\phi,\theta] = [\sqrt{69}, \cos^{-1}(-8/\sqrt{69}), \tan^{-1}(2)]$$

$$\approx [8.31, 2.87, 1.11]$$

$$r = \sqrt{x^2+y^2} = \sqrt{1+4} = \sqrt{5} \approx 2.24$$

Sylinderkoordinater:

$$[r,\theta,z] = [\sqrt{5}, \tan^{-1}(2), -8]$$

$$\approx [2.24, 1.11, -8]$$

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3 (c) (ii) $\vec{r}_0 = \vec{i} + \vec{j}$, $\vec{r}_1 = \vec{j} + \vec{k}$, $\vec{v} = \vec{i} - \vec{j} - \vec{k}$

$$\boxed{s = \frac{|(\vec{r}_0 - \vec{r}_1) \times \vec{v}|}{|\vec{v}|}}$$

$$\vec{r}_0 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad \vec{r}_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

$$(\vec{r}_0 - \vec{r}_1) \times \vec{v} = \left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right) \times \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \times -1 \\ -1 \times -1 \\ 1 \times 1 \\ 0 \times -1 \end{bmatrix} = \begin{bmatrix} 0 \cdot (-1) - (-1) \cdot (-1) \\ (-1) \cdot 1 - (-1) \cdot 1 \\ 1 \cdot (-1) - 1 \cdot 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$$

$$s = \frac{\left\| \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix} \right\|}{\left\| \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} \right\|} = \frac{\sqrt{2}}{\sqrt{3}} = \sqrt{\frac{2}{3}} \approx 0.82$$

③ (d)(i) $A=(1,4,-5)$ $B=(3,6,-2)$ $C=(-3,-2,3)$

retningsVektor $\vec{v}$ langs linje gjennom A og B:

$$\vec{v} = \begin{bmatrix} 3-1 \\ 6-4 \\ -2-(-5) \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = a\vec{i} + b\vec{j} + c\vec{k}$$

velger A som $P_0 = (x_0, y_0, z_0)$

Standardform: $$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$$

$$\frac{x-1}{2} = \frac{y-4}{2} = \frac{z+5}{3}$$

③ (d) (ii) $A=(1,4,-5)$ $B=(3,6,-2)$ $C=(-3,-2,3)$

$\vec{V}_1$ = vektor fra A til B

$\vec{V}_2$ = vektor fra A til C

$\vec{n}_1$ = kryssprodukt mellom $\vec{v}_1$ og $\vec{v}_2$

$P_0 = A$ = punkt planet går gjennom

$$\vec{V}_1 = \begin{bmatrix} 3-1 \\ 6-4 \\ -2-(-5) \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} \qquad \vec{V}_2 = \begin{bmatrix} -3-1 \\ -2-4 \\ 3-(-5) \end{bmatrix} = \begin{bmatrix} -4 \\ -6 \\ 8 \end{bmatrix}$$

$$\vec{V}_1 \times \vec{V}_2 = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} \times \begin{bmatrix} -4 \\ -6 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \times -6 \\ 3 \times 8 \\ 2 \times -4 \\ 2 \times -6 \end{bmatrix} = \begin{bmatrix} 16-(-18) \\ -12-16 \\ -12+8 \end{bmatrix} = \begin{bmatrix} 34 \\ -28 \\ -4 \end{bmatrix}$$

$$\vec{n}_1 = \begin{bmatrix} 34 \\ -28 \\ -4 \end{bmatrix}$$

siden $\vec{n}_1$ har faktor 2 i alle komponenter, kan vi velge $\vec{n} = \frac{\vec{n}_1}{2}$

$$\vec{n} = \begin{bmatrix} 17 \\ -14 \\ -2 \end{bmatrix}$$

$$17(x-1) - 14(y-4) - 2(z+5) = 0$$

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