Tidligere eksamensoppgaver R2 - Geometri

DEL 1 - OPPGAVER

Eks. 08-1-g:

$A(1,1,1) \quad B(3,3,2) \quad C(2,1,2)$

$$\vec{AB} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \qquad \vec{AC} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Fra oppgitt tabell

$$u = \cos^{-1}\left(\frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| \cdot |\vec{AC}|}\right) = \cos^{-1}\left(\frac{2+0+1}{\sqrt{9} \cdot \sqrt{2}}\right) = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = 45°$$

Eks. V09-1-d:

$A(2,3,7) \quad B(3,5,2) \quad C(1,1,5) \quad D(3,5,t)$

1) $$\vec{AB} = \begin{bmatrix} 1 \\ 2 \\ -5 \end{bmatrix} \qquad \vec{AD} = \begin{bmatrix} 1 \\ 2 \\ t-7 \end{bmatrix} \qquad \vec{AB} \cdot \vec{AD} = 0 \Leftrightarrow \vec{AB} \perp \vec{AD}$$

$$\begin{bmatrix} 1 \\ 2 \\ -5 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ t-7 \end{bmatrix} = 1 + 4 + -5t + 35 = -5t + 40 = 0 \Rightarrow t = 8$$

2) $\vec{AB} = k \cdot \vec{AD} \Rightarrow \vec{AB} \parallel \vec{AD}$:

$$\begin{bmatrix} 1 \\ 2 \\ -5 \end{bmatrix} = k \cdot \begin{bmatrix} 1 \\ 2 \\ t-7 \end{bmatrix} \Rightarrow \begin{aligned} 1 &= k & &\Rightarrow k = 1 \\ 2 &= 2k & &\Rightarrow k = 1 \\ -5 &= tk - 7k & &\Rightarrow t = 2 \end{aligned}$$

Eks. V09-2:

$A(1,0,0) \quad B(0,2,2) \quad C(1,1,2) \quad D(4,1,-3)$

a) $\vec{AB} = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} \qquad \vec{AC} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \qquad \vec{AB} \times \vec{AC} = \begin{bmatrix} 2 & 1 \\ 2 & 2 \\ -1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 4-2 \\ 0+2 \\ -1-0 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$

Areal: $\frac{1}{2} \cdot \left| \vec{AB} \times \vec{AC} \right| = \frac{1}{2}\sqrt{4+4+1} = \frac{3}{2}$

b) Volum: $\frac{1}{6} \cdot \left| (\vec{AB} \times \vec{AC}) \cdot \vec{AD} \right| = \frac{1}{6} \left| \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \\ -3 \end{bmatrix} \right| = \frac{1}{6} \left| 6+2+3 \right| = \frac{1}{6} \cdot 11 = \frac{11}{6}$

c) $n_\alpha = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$; $2(x-1) + 2(y-0) + -1(z-0) = 0$

$$\Downarrow$$

$\alpha: \quad 2x - 2 + 2y - z = 0$

Eks. H10-2:

$A(1,1,1) \quad B(3,2,3) \quad C(2,7,5)$

a) $\vec{AB} \cdot \vec{AC} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix} = 2 + 6 + 8 = 16$

b) $\vec{AB} \times \vec{AC} = \begin{bmatrix} 1 & 6 \\ 2 & 4 \\ 2 & 1 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 4-12 \\ 2-8 \\ 12-1 \end{bmatrix} = \boxed{\begin{bmatrix} -8 \\ -6 \\ 11 \end{bmatrix}}$

c) $\vec{n}_\alpha = \begin{bmatrix} -8 \\ -6 \\ 11 \end{bmatrix}$; $-8(x-1) - 6(y-1) + 11(z-1) = 0$

$-8x + 8 - 6y + 6 + 11z - 11 = 0 \qquad | \cdot (-1)$

$\alpha: \ 8x + 6y - 11z - 3 = 0$

$D(2,2,3)$ i α?: $8 \cdot 2 + 6 \cdot 2 - 11 \cdot 3 - 3 = 16 + 12 - 33 - 3 = 28 - 36 \neq 0 \Rightarrow$ NEI

Eks H10-2:

d) $\vec{r}_l = -\vec{n}_\alpha = \begin{bmatrix} 8 \\ 6 \\ -11 \end{bmatrix}$; $l: \begin{cases} x = 2 + 8t \\ y = 2 + 6t \\ z = 3 - 11t \end{cases}$

Skjæring mellom l og α: $8(2+8t) + 6(2+6t) - 11(3-11t) - 3 = 0$

$$16 + 64t + 12 + 36t - 33 + 121t - 3 = 0$$

$$221t + 28 - 36 = 0$$

$$221t = 8 \Rightarrow t = \frac{8}{221}$$

$$\Rightarrow S\left(2 + 8 \cdot \frac{8}{221}, 2 + 6 \cdot \frac{8}{221}, 3 - 11 \cdot \frac{8}{221}\right)$$

$$= S\left(\frac{442+64}{221}, \frac{442+48}{221}, \frac{663-88}{221}\right)$$

$$= S\left(\frac{506}{221}, \frac{490}{221}, \frac{575}{221}\right)$$ ← Dette er riktig svar, men det er rart at de har laget oppgaven slik.

Eks V10-2:

$A(3,0,-2)$ $B(0,2,0)$ $C(1,-1,4)$

a) $\vec{AB} \times \vec{AC} = \begin{bmatrix} -3 \\ 2 \\ 2 \end{bmatrix} \times \begin{bmatrix} -2 \\ -1 \\ 6 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 2 & 6 \\ -3 & -2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 12+2 \\ -4+18 \\ 3+4 \end{bmatrix} = \begin{bmatrix} 14 \\ 14 \\ 7 \end{bmatrix}$

b) $\vec{n}_\alpha = \frac{1}{7}\begin{bmatrix} 14 \\ 14 \\ 7 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$ $\alpha: 2(x-0) + 2(y-2) + 1(z-0) = 0$

$\alpha: 2x + 2y + z - 4 = 0$

c) $\vec{r}_l = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$, $P_0(5,4,4)$ gir $l: \begin{cases} x = 5 + 2t \\ y = 4 + 2t \\ z = 4 + t \end{cases}$

x-z-planet: $y = 0$

Skjæring mellom l og x-z-planet: $4 + 2t = 0 \Rightarrow t = -2$

$$\Rightarrow (5 + 2\cdot(-2), 0, 4-2)$$

$$= (1, 0, 2)$$

d) $Q(5+2t,\ 4+2t,\ 4+t)$

$$\vec{AQ} = \begin{bmatrix} 5+2t-3 \\ 4+2t-0 \\ 4+t+2 \end{bmatrix} = \begin{bmatrix} 2t+2 \\ 2t+4 \\ t+6 \end{bmatrix}$$

Volum: $\frac{1}{6} \cdot \left| (\vec{AB} \times \vec{AC}) \cdot \vec{AQ} \right| = \frac{1}{6} \left| \begin{bmatrix} 14 \\ 14 \\ 7 \end{bmatrix} \cdot \begin{bmatrix} 2t+2 \\ 2t+4 \\ t+6 \end{bmatrix} \right|$

$$= \frac{1}{6} \left| 14(2t+2) + 14(2t+4) + 7(t+6) \right|$$

$$= \frac{1}{6} \left| 28t + 28 + 28t + 56 + 7t + 42 \right|$$

$$= \frac{1}{6} \left| 63t + 126 \right| = \frac{1}{6} \left| 21 \cdot 3 \cdot t + 2 \cdot 3 \cdot 21 \right| = \frac{1}{2} \left| 21t + 42 \right|$$

$$V = \left| \frac{21}{2}t + 21 \right|$$

e) $V = \left| \frac{21}{2}t + 21 \right| = 42$

$$\frac{21}{2}t + 21 = \pm 42$$

$$\frac{21}{2}t = -21 \pm 42$$

$$21t = -42 \pm 84$$

$$t = -2 \pm 4 = 2 \text{ eller } -6$$

$Q_0(5+2\cdot 2,\ 4+2\cdot 2,\ 4+2)$ eller $Q_1(5+2\cdot(-6),\ 4+2\cdot(-6),\ 4-6)$

$= Q_0(9, 8, 6)$ eller $Q_1(-7, -12, -2)$

Eks. H10-1-d:

$A(1,0,3) \quad B(3,2,4) \quad C(5,3,0)$

1) $|\vec{AB}| = \left\|\begin{bmatrix}2\\2\\1\end{bmatrix}\right\| = \sqrt{4+4+1} = 3$

$|\vec{AC}| = \left\|\begin{bmatrix}4\\3\\-3\end{bmatrix}\right\| = \sqrt{16+9+9} = \sqrt{34}$

2) $\vec{AB}\cdot\vec{AC} = \begin{bmatrix}2\\2\\1\end{bmatrix}\cdot\begin{bmatrix}4\\3\\-3\end{bmatrix} = 8+6-3 = 11$

$\vec{AB}\times\vec{AC} = \begin{bmatrix}2 & 3\\1 & -3\\2 & 4\\2 & 3\end{bmatrix} = \begin{bmatrix}-6-3\\4+6\\6-8\end{bmatrix} = \begin{bmatrix}-9\\10\\-2\end{bmatrix}$

3) $|\vec{AB}\times\vec{AC}|^2 = \left\|\begin{bmatrix}-9\\10\\-2\end{bmatrix}\right\|^2 = \sqrt{9^2+10^2+2^2}^2 = 9^2+10^2+2^2 = 81+100+4 = 185$

$(\vec{AB}\cdot\vec{AC})^2 = 11^2 = 121$

$|\vec{AB}|^2\cdot|\vec{AC}|^2 = 3^2\cdot\sqrt{34}^2 = 9\cdot 34 = \boxed{3^2\cdot 2\cdot 17}$

Like?

- - - -

$|\vec{AB}\times\vec{AC}|^2 + (\vec{AB}\cdot\vec{AC})^2 = 185+121 = 306 = 3\cdot 102 = \boxed{3\cdot 2\cdot 51}$

$51 = 3\cdot 17$?

$$\begin{array}{r} {}^{2}17 \\ 17 \\ 17 \\ \hline 51 \end{array}$$

JA! Det stemmer.